



Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

1982

Calculation of the longitudinal stability
derivations and modes of motion for
helicopter aircraft.

O'Neill, Hugh James.

Monterey, California. Naval Postgraduate School

<http://hdl.handle.net/10945/20089>

Downloaded from NPS Archive: Calhoun



<http://www.nps.edu/library>

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIF. 93940

NAVAL POSTGRADUATE SCHOOL

Monterey, California



THESIS

CALCULATION OF THE LONGITUDINAL STABILITY
DERIVATIVES AND MODES OF MOTION
FOR HELICOPTER AIRCRAFT

by

Hugh James O'Neill

October 1982

Thesis Advisor:

D. M. Layton

Approved for public release; distribution unlimited

T206937

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Calculation of the Longitudinal Stability Derivatives and Modes of Motion for Helicopter Aircraft		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis October 1982
7. AUTHOR(s) Hugh James O'Neill		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE October 1982
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES 113
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Longitudinal Stability Short Period Phugoid Modes of Motion Dynamic Motion Characteristic Equation		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This thesis presents an analysis of the longitudinal stability derivatives for helicopter aircraft and is intended to be used as a resource document for a helicopter stability and control course at the Naval Postgraduate School. Emphasis is given to the evolution of forces and moments on the helicopter, calculation of the stability derivatives at high advance ratios, derivation of the stability determinant and solution of the characteristic equation to yield the modes of motion of the helicopter.		

Approved for public release; distribution unlimited

Calculation of the Longitudinal Stability Derivatives
and Modes of Motion for Helicopter Aircraft

by

Hugh James O'Neill
Major, United States Marine Corps
B.S., United States Naval Academy, 1969

Submitted in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
October 1982

ABSTRACT

This thesis presents an analysis of the longitudinal stability derivatives for helicopter aircraft and is intended to be used as a resource document for a helicopter stability and control course at the Naval Postgraduate School.

Emphasis is given to the evolution of forces and moments on the helicopter, calculation of the stability derivatives at high advance ratios, derivation of the stability determinant and solution of the characteristic equation to yield the modes of motion of the helicopter.

TABLE OF CONTENTS

I.	INTRODUCTION -----	6
	A. EQUILIBRIUM -----	7
	B. STATIC STABILITY -----	7
	C. DYNAMIC STABILITY -----	10
	D. AXES SYSTEMS -----	12
	1. Gravity Axis -----	14
	2. Stability Axis -----	15
	3. Body Axis -----	16
II.	THE EQUATIONS OF MOTION -----	17
	A. LINEAR MOTION -----	17
	B. ANGULAR MOTION -----	21
	C. FORCES AND MOMENTS -----	26
	D. CORIOLIS FORCES AND MOMENTS -----	33
	E. RIGID BODY EQUATIONS -----	37
	1. Three Force Equations -----	40
	2. Three Moment Equations -----	40
	F. EXPANSION OF FORCES AND MOMENTS -----	40
III.	CALCULATION OF THE STABILITY DERIVATIVES -----	43
	A. TYPES AND USES -----	43
	B. DEFINITION OF TERMS -----	46
	C. HELICOPTER DATA -----	48
	D. CALCULATION OF THE HELICOPTER TRIM VALUES -----	49
	E. CALCULATION OF THE ROTOR ISOLATED DERIVATIVES -----	57

F.	CALCULATION OF THE LOCAL DERIVATIVES -----	59
G.	CALCULATION OF THE TOTAL STABILITY DERIVATIVES -----	62
IV.	SOLUTION OF THE CHARACTERISTIC EQUATION -----	66
A.	THE CHARACTERISTIC EQUATION -----	68
B.	HELICOPTER RESPONSE IN A HOVER -----	71
C.	APPROXIMATION OF THE SHORT PERIOD DURING HOVER -----	73
D.	HELICOPTER RESPONSE IN FORWARD FLIGHT -----	74
E.	APPROXIMATION OF THE SHORT PERIOD IN FORWARD FLIGHT ----	77
V.	INTERPRETATION OF THE STABILITY DERIVATIVES -----	78
VI.	ANALYSIS OF RESULTS -----	82
APPENDIX A.	SAMPLE PROBLEM -----	85
APPENDIX B.	COMPUTER PROGRAMS FOR THE HP-41CV CALCULATOR -----	101
LIST OF REFERENCES	-----	111
INITIAL DISTRIBUTION LIST	-----	112

I. INTRODUCTION

The primary interest of any aviator engaged in flight is control of his aircraft. Without this, it becomes impossible to maintain the aircraft in a desired flight regime and consequently the flight is rather abruptly terminated. With adequate control over his vehicle however, the aviator is capable of successfully performing many various tasks to include a safe final landing.

Historically, the development of the helicopter was hampered during its early years of evolution because of the lack of understanding of the factors which caused stability (or instability) and therefore adequate control over the vehicle was difficult to achieve. This situation was exactly similar to that of the early development of the airplane. It was only when the principles of stability and control could be understood that aircraft could be developed which would fly as the designer intended them to fly and that helicopter development could progress.

Before the details of controlling the aircraft can be fully worked out, some understanding of the aircraft's inherent stability must be attained because these two factors, stability and control, are closely related. Together they determine the flying qualities, or handling qualities of the aircraft. The aircraft must have sufficient stability to maintain a certain desired condition of flight and to recover normally from disturbing influences (wind gusts, for instance). Pilot workload is also a function of stability. Since adequate maneuverability is a necessity, the aircraft must also respond properly to the pilot's inputs.

An expansion of the concepts behind aircraft stability will be discussed here so that they can be more fully understood and easily dealt with when moving into the area of aircraft control.

A. EQUILIBRIUM

A helicopter is in a state of equilibrium when the vector sum of all forces and moments on it are equal to zero. While in equilibrium the aircraft will not have any tendencies to accelerate in either translational (no unbalanced forces) or rotational (no unbalanced moments) directions. Thus the aircraft will remain in a steady flight condition.

If, however, forces or moments are introduced to upset this balanced condition (via cockpit control inputs or wind gusts, for example) the helicopter will experience an acceleration in the direction of the unbalanced forces and/or moments. As can be expected from Newton's Second Law, linear accelerations are proportional to the magnitude of the unbalanced forces and the angular accelerations are proportional to the unbalanced moments.

B. STATIC STABILITY

The static stability of a system is defined by the initial tendency of the system to return to equilibrium conditions following some disturbance from equilibrium. If an object which is disturbed from equilibrium tends to return to equilibrium, the object has positive static stability. On the other hand, if the object, upon being disturbed, has a tendency to continue in the direction of the disturbance, then the object is exhibiting behavior of negative static stability. Neutral

static stability exists when the object has no tendency to return to equilibrium or to continue in the direction of the disturbance.

The classic physical examples of static stability are shown in Figure I-1 [Ref. 1]. Various tendencies of motion of a ball displaced

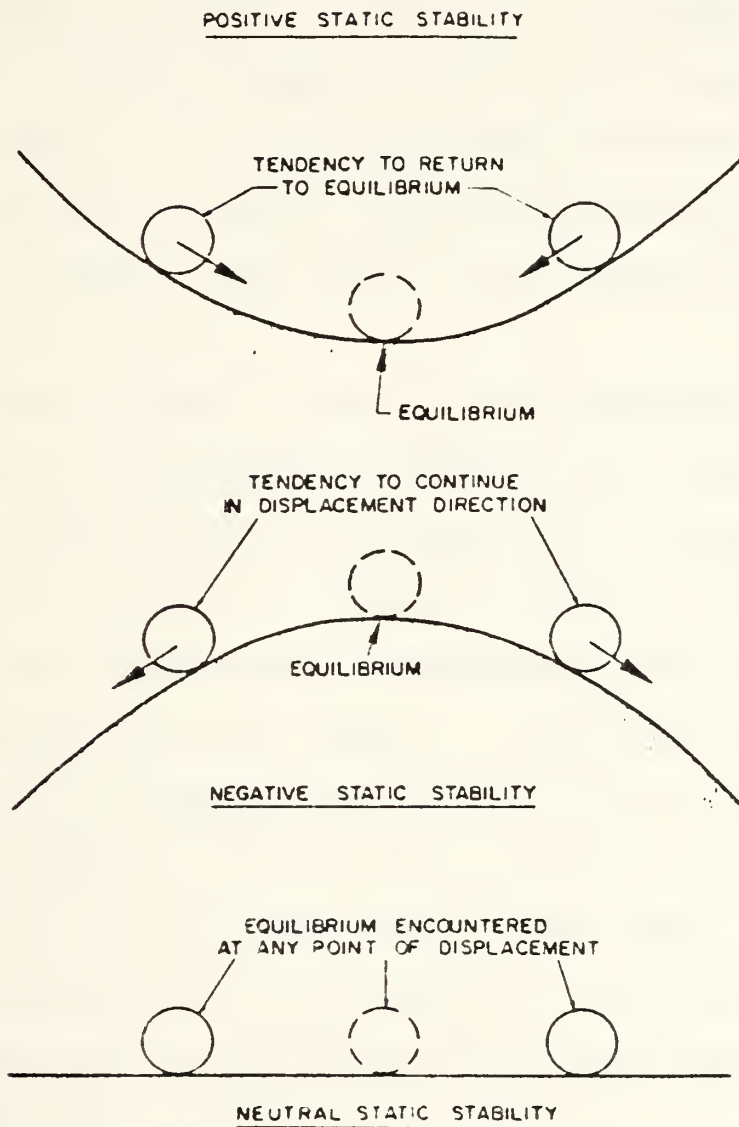


Figure I-1. Static Stability

from equilibrium in a depression, on a hill, and on a level surface are shown. It should be noted that positive static stability is the desired response in most situations.

It should also be noted that there are quantitative degrees of static stability. This has to do with the forces acting on an object after it has been disturbed from the equilibrium condition. An example will illustrate this point. A large ball in a shallow depression may have a force of one-half pound returning it to the equilibrium position while the same size ball in a very steep-sided depression may have a restoring force of ten pounds. While both these systems exhibit positive static stability, the second is more positive and thus a more stable system.

If a stability control system is to be incorporated into the helicopter then the magnitude of the aircraft's static stability terms will be one clue to the amount of force the control system must have to be effective. For example, if a system has negative static stability in yaw, some control feature must be incorporated to allow the pilot to keep the yawing motion under control. If the negative yaw static stability term is of large magnitude, then the control system will have to have a great amount of power to control the tendency of the aircraft to diverge in yaw. On the other hand, if the stability term is only slightly negative, then the yawing motion will not diverge as quickly as in the former case nor will the controlling power needed be as great as in the former case.

For a complete discussion of helicopter control, dynamic stability must also be included.

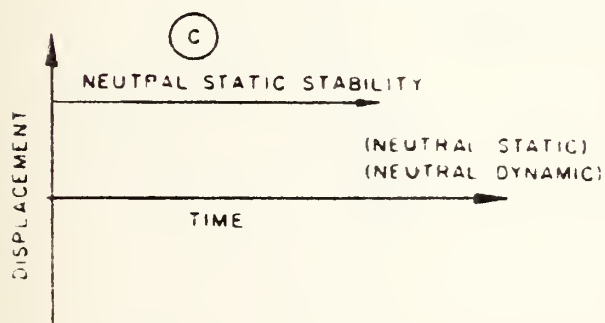
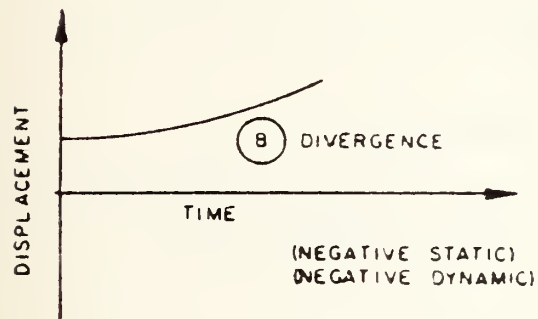
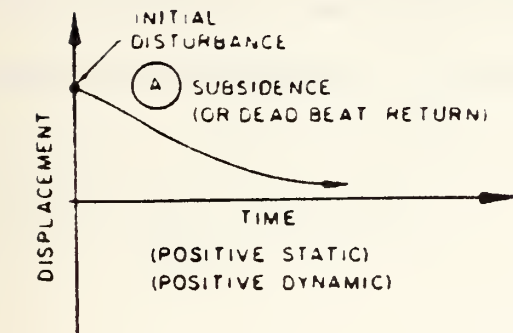
C. DYNAMIC STABILITY

Dynamic stability refers to a body's resulting motions with respect to time after being disturbed from equilibrium. A plot of displacement versus time will reveal the dynamic stability tendencies of a body. All possible responses of a disturbed body can be seen in Figure I-2. Two general modes of motion exist, oscillatory and non-oscillatory (also called periodic and aperiodic). As is implied by the term, oscillatory, the position of the body will cycle in some manner about the equilibrium position.

The motion of both the oscillatory and non-oscillatory modes will also depend on whether or not such motion is damped. If a body eventually returns to the equilibrium position, then its motion is said to be damped. If insufficient damping is available, then the body's motion will become divergent. Divergent oscillations are generally undesirable and usually result in material failure of a mechanical system. Damping tends to take energy out of a system. Some factors which cause damping are friction, hydraulic dampers, springs, etc. Divergent motions have energy added to the dynamic system. An example of this is a pilot-induced oscillation (PIO) where the pilot's control movements are in the same direction and with the same timing as the aircraft's response.

Neutral stability is another possible behavior of a disturbed body. Here the body remains at its original disturbed state or oscillates at a constant amplitude about the equilibrium state. Static stability is necessary for dynamic stability to exist, but the converse is not true.

NON-OSCILLATORY MODES



OSCILLATORY MODES

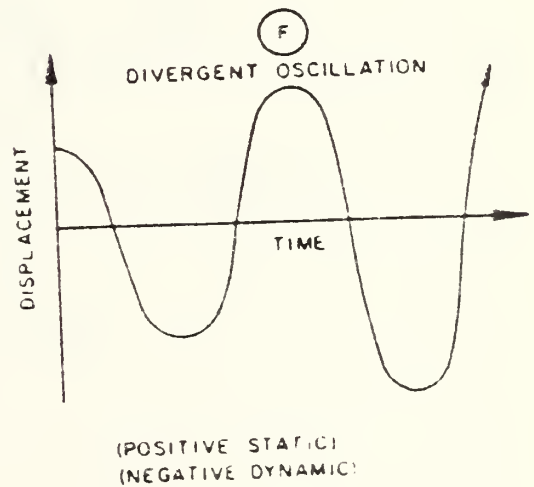
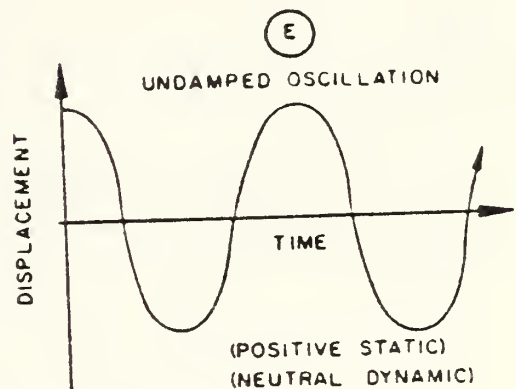
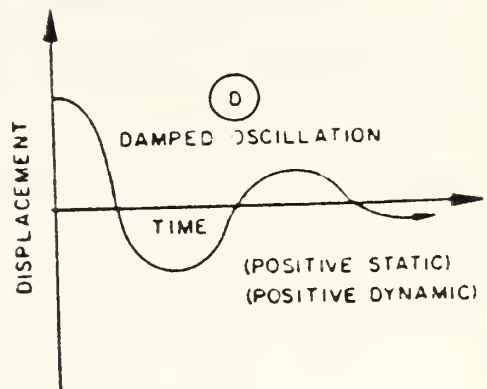


Figure I-2. Dynamic Stability

D. AXES SYSTEMS

To establish a basis for the discussion of unbalanced forces and moments on an aircraft and that aircraft's subsequent reaction, a reference coordinate frame must be established. Figure I-3 illustrates the conventional arrangement of perpendicular axes which are centered at the helicopter's center of gravity. The directions indicated by the arrows are the positive direction.

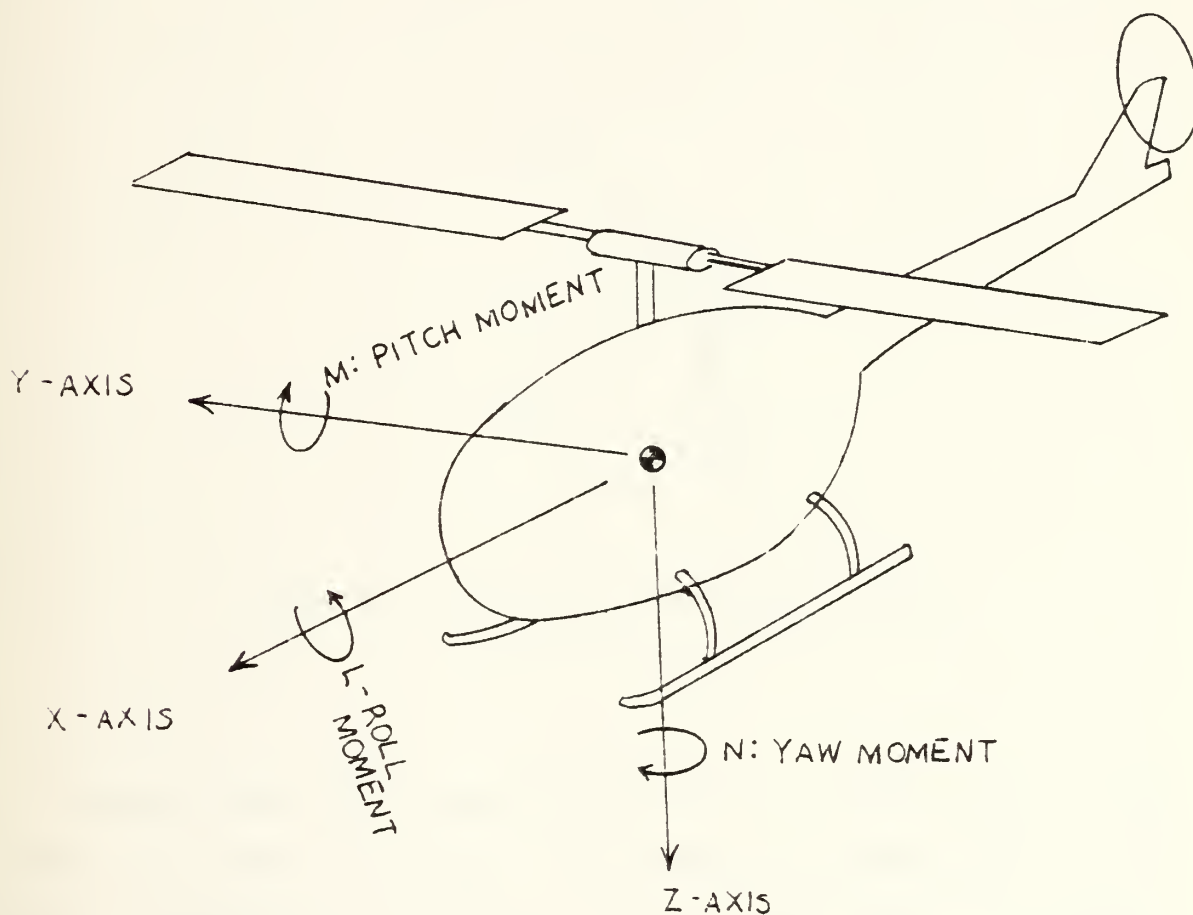


Figure I-3. Axis System Notation

The positive directions of this axis system for the X, Y, and Z axes are forward, right, and down, respectively. Thus this is a right-handed system. (Note: A right-handed rectangular coordinate system derives its name from the analogy that a right-threaded screw rotated through 90 degrees from OX to OY will advance in the positive Z direction (see Figure I-4). Forces are named for the directions along which they act. Thus an X-force is one acting in the X-direction. The same nomenclature system applies for the Y and Z forces.

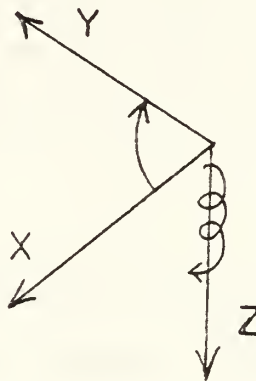


Figure I-4. Right-Handed Coordinate System

Rational motion also occurs about the X, Y, and Z axes. These moments are termed L, M, and N. L is the rolling moment which occurs about the longitudinal (X) axis. A roll to the right is defined as positive. M is the pitching moment which occurs about the lateral (Y) axis. A nose-up pitch describes a positive value of M. The yawing

N-moment occurs about the vertical (Z) axis. A positive yaw is defined as one which moves the nose of the helicopter to the right. Table I-1 summarizes the axis system notation.

TABLE I-1. Axis System Notation

AXIS	FORCE	VELOCITY	MOMENT	ANGULAR MEASURE	ANGULAR VELOCITY
X	X	U	L	ϕ	P
Y	Y	V	M	θ	Q
Z	Z	W	N	β	R

This same set of orthogonal axes can be referenced in various ways, depending on the needs of the engineer. Certain particular problems dealing with aircraft stability can be solved more easily by the proper selection of axis reference.

Three systems of axis reference are generally used:

- (1) Gravity Axis,
- (2) Stability Axis, and
- (3) Body Axis.

1. Gravity Axis

In this system the Z-axis of the helicopter is always pointing at the center of the earth, and the X-axis is directed along the horizon. The gravity axis system is useful for linear displacements and angular

accelerations. Certain simplifications in the stability derivatives can be achieved; however when helicopter rotation is taken into account inertial terms and products of inertia must have lengthy corrections.

2. Stability Axis

In this system of reference, the X-axis is aligned with the velocity vector and is positive pointing into the relative wind. The Z-axis is perpendicular to the relative wind and the Y-axis is orthogonal to both, forming a right-hand system. Using the stability axis system can yield great simplifications of the aerodynamic terms. This system is limited to small disturbance motions, however, because the moment of inertia terms vary and thus are assumed to be constant in the equations of motion.

The stability axis reference system has the useful feature of being directly applicable to wind tunnel results which are commonly measured parallel with and perpendicular to the wind.

The stability axis is very useful for fixed-wing analysis, because the relative wind is always somewhat directly on the nose of the aircraft and varies very little from that direction. The unique capability of a helicopter to decelerate from forward flight to hovering flight (zero forward velocity) means that the X- and Z-axes could change by as much as 90 degrees as the relative velocity of the air changes from horizontal (on the nose of the helicopter) to vertical (being pulled down through the rotor disk). Thus the usefulness of this axis system for helicopters is limited since it cannot be used for comparison purposes over the whole range of the helicopter's velocity.

3. Body Axis

The body axis system aligns the X-axis with a datum line on the helicopter. The Z-axis is perpendicular to the X-axis and is directed out the bottom of the aircraft while the Y-axis is orthogonal to both. This system ensures that the inertial terms in the equations of motion are independent of the flight conditions.

The reference system of body axes is very useful when studying helicopter dynamics because velocities and accelerations with respect to these axes are the same as those that would be measured by instruments in the helicopter and those that are experienced by the pilot.

II. THE EQUATIONS OF MOTION

To obtain solutions of aircraft stability, some quantitative data must be made available to the engineer. Naturally some formulae or equations would be helpful when trying to arrive at a mathematical determination of the stability problem at hand.

In this chapter some elementary concepts will be introduced which will lead to the development of the equations of motion of the flight vehicle. It is these equations of motion which will yield the numerical data needed for problem solutions.

A. LINEAR MOTION

Linear motion is the motion of an object along a line. The line can either be straight or curved so that linear motion can be further subdivided in rectilinear (straight line) or curvilinear (along a curved line) motion.

1. Rectilinear Motion

The rectilinear motion of a particle can be described by that particle's position on a straight line and its time derivatives of position. To quantify this motion a reference point must be selected. All subsequent measurements of the particle's motion are made with respect to this reference point. A coordinate system well suited for rectilinear motion is the rectangular cartesian coordinate system.

By making use of a selected coordinate system, p marks the generalized coordinates of the particle's position. The distance, s , of

the particle from the selected reference point is the difference of the coordinates for these two points. If the origin of the coordinate system is chosen as the reference point, then the distance to the particle simply becomes the value p . If some point other than the origin is chosen as the reference point, then the distance of the particle from the reference point is $p - r$ where r is the coordinate of the reference point from the origin. Should the particle move to a new point on the line, p' , it would then be at a different distance from the origin. This difference is Δs (see Figure 2-1).

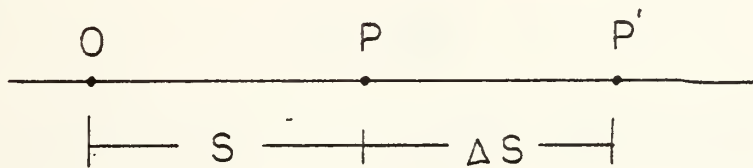


Figure 2-1. Measurement Along a Line

The distance is given as:

$$\Delta s = p - p' \quad (2-1)$$

The rate at which the particle travels from p to p' is very often of interest. Thus the average velocity of the particle along this path is the distance along the path divided by the time it took to cover the distance.

$$v_{avg} = \frac{\Delta s}{\Delta t} \quad (2-2)$$

As the time interval becomes very small, the result will be the instantaneous velocity of the particle along the path.

$$v = \lim_{t \rightarrow 0} \frac{\Delta s}{\Delta t} \quad (2-3)$$

Both average and instantaneous velocities will be used in subsequent calculations.

Similarly, it can be shown that the acceleration of the particle is the rate of change of the particle's velocity.

$$a_{avg} = \frac{\Delta v}{\Delta t} \quad (2-4)$$

In the same manner as for velocity, the instantaneous acceleration of the particle can be shown to be:

$$a = \lim_{t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \dot{v} \quad (2-5)$$

and since $dv = ds/dt$, acceleration can also be expressed as the second derivative of position with respect to time.

$$a = \frac{ds/dt}{dt} = \frac{d^2s}{dt^2} = \ddot{s} \quad (2-6)$$

The calculation of accelerations become important because they lead directly to forces on the airframe via Newton's Second Law:

$$F = ma \quad (2-7)$$

2. Curvilinear Motion

The second type of linear motion occurs along a curved path and is therefore called curvilinear motion. Plane curvilinear motion occurs when a particle that is moving along a curved path remains in a single plane. For this case the position in which the particle motion occurs (see Figure 2-2).

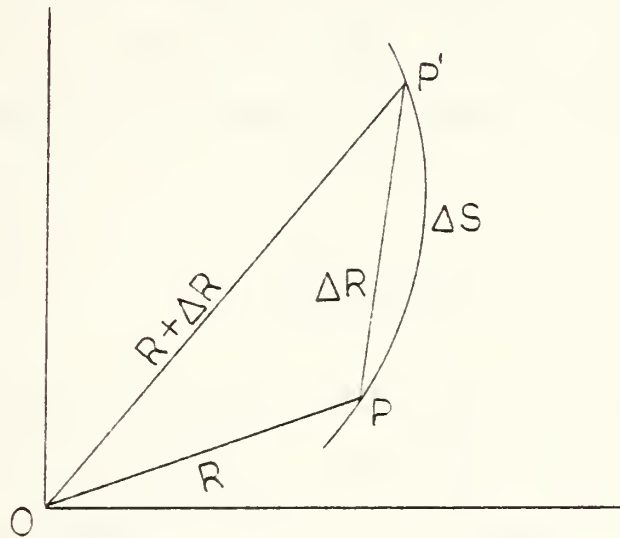


Figure 2-2. Curvilinear Motion

In this figure, the particle has moved from point p to point p' along the curved path. The particle's motion takes place in the plane of the paper and occurs about point O . The position of the new point p' is

given by the vector addition of the old position vector at p (vector r) plus the change that occurs during the movement (vector Δr).

As was the case with rectilinear motion, the average velocity of a point along the path is equal to the time rate of change of the point's position vector. Thus

$$v_{\text{avg}} = \frac{\Delta r}{\Delta t} \quad (2-8)$$

If the time interval becomes smaller and smaller, the Δr vector approaches tangency to the curved path. In the limiting case, the velocity will approach the instantaneous velocity of the particle along the curved path.

$$v = \lim_{t \rightarrow 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \quad (2-9)$$

If the speed of the particle increases as it moves from p to p', then the particle is accelerating. The same steps that were used to develop the instantaneous velocity can be followed to find the acceleration term. The result is

$$a = \lim_{t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 r}{dt^2} \quad (2-10)$$

B. ANGULAR MOTION

While linear motion can be used to describe a significant number of motions that are commonly encountered in physics, motion can also be described in another way. Angular measurements to a moving point from a fixed point are commonly used when dealing with rotating systems.

The description of angular motion will be limited to that occurring in a plane. Before this angular motion can be described, some reference point must be selected. A point in the plane of motion is a good reference point and an axis perpendicular to the plane of motion serves as the axis about which the angular motion occurs. This becomes the axis of rotation.

Before angular measurement can begin, a reference axis in the plane of rotation must be chosen. All angular positions will be measured with respect to this reference axis. The reference axis itself is chosen arbitrarily and it does not rotate. Although the system being measured may rotate, the reference axis remains fixed.

Angular displacements are measured in degrees or radians using the symbol θ . By convention, a positive angular displacement is in a counterclockwise direction from the reference axis (see Figure 2-3).

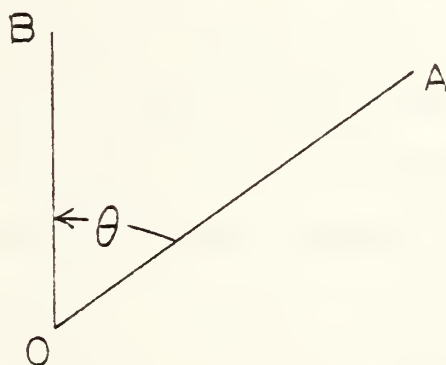


Figure 2-3. Angular Measure

According to the foregoing rules, the angular measurement of line OB is at some angle θ from the arbitrarily chosen reference axis, OA. The axis of rotation is perpendicular to the paper at point O. The angular rate at which the line moves from position OA to OB in Figure 2-3 is the time rate of change of its angular position. As was the case with previous descriptions of linear velocity, the angular velocity, ω , is the first time derivative of the angular position.

$$\omega = \frac{d\theta}{dt} = \dot{\theta} \quad (2-11)$$

Similarly, angular acceleration, α , is the time derivative on angular velocity or the second time derivative of angular position.

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} = \ddot{\theta} \quad (2-12)$$

A common case of motion combines both curvilinear and angular motions. This case of motion occurs when a particle moves around a fixed point at a fixed distance from that point. An example of this is the arc traced out by the helicopter's rotor blade tips.

As a helicopter blade sweeps the air, all points on the blade are rotating at the same angular velocity. However each point at a different radius from the center of the axis of rotation has a different curvilinear velocity or speed. This can be readily understood by examining Figure 2-4. R_1 and R_2 are two points located on the same rotating line (for example, a rotor blade). During a single rotation of the rotor disk, R_2 travels a greater distance than R_1 . The distance traveled by each point is the circumference of the circle traced out by

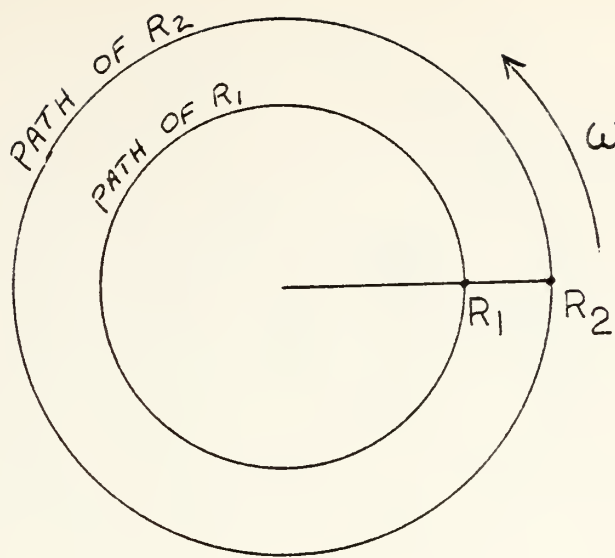


Figure 2-4. Dependence of Linear Velocity on Radius

the point. The path traced out by R_2 is longer than that traced out by R_1 . Since both points complete one revolution of the circle in the same amount of time, R_2 must travel faster than R_1 .

The velocity of a particle rotating about a fixed point is given by both its angular velocity and distance from the axis of rotation and is developed in the following manner (refer to Figure 2-5). The instantaneous velocity of the rotating point is the limit as time approaches zero of the change in the particle's position vector divided by the average time over which this position change occurs.

$$v = \lim_{t \rightarrow 0} \frac{\Delta r}{\Delta t} \quad (2-13)$$

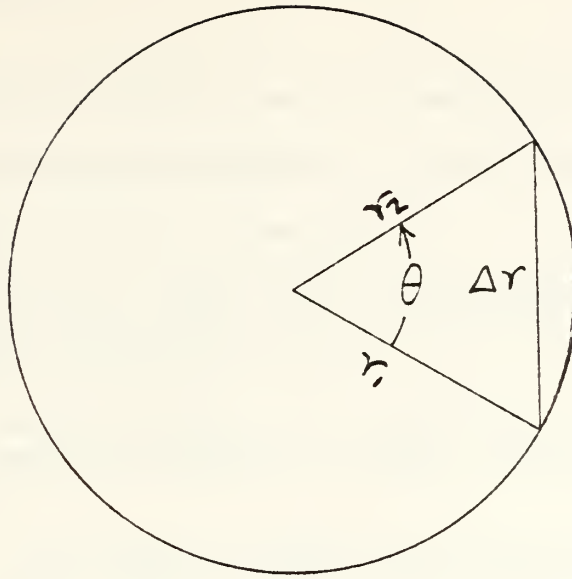


Figure 2-5. Linear Velocity of a Rotating Point

where $\Delta r = r$ times the change in angular displacement. Thus

$$v = \lim_{t \rightarrow 0} \frac{r \Delta \theta}{\Delta t} = r\omega \quad (2-14)$$

r is measured in feet or meters and ω is measured in radians per second.

It can be seen from equation (2-14) that a particle located at half the distance from the rotor hub to the tip of the rotor is moving at half the linear velocity of a particle located at the rotor tip.

C. FORCES AND MOMENTS

1. Forces

No discussion of forces would be complete without giving credit to Sir Issac Newton and his statements of the basic laws governing the motion of a particle [Ref. 2]. They are:

Law I. A particle remains at rest or continues to move in a straight line with a uniform velocity if there is no unbalanced force acting on it.

Law II. The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of the force.

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

The word force is mentioned in all three of Newton's laws. Therefore it would seem that force plays a large part in the movement of a particle (or of an object, if the center of gravity of that object is considered to be a particle). Force can be defined as the action of one body on another. A force acting in a direction tends to move an object it acts upon in the same direction, according to Law II.

It should be noted that forces are vector quantities. That is, they are composed of a magnitude and a direction. The direction of a force becomes very important when attempting to predict the reactions of objects to forces.

The importance of the vectorial nature of forces cannot be over-emphasized. For example, what is the resultant of a 50-pound force applied to another 50-pound force? The correct answer could be zero pounds, 100 pounds, or anywhere in between depending on whether the

forces were directly opposite to each other, in the same direction, or at some other angular position between these two extremes.

Newton's second law is of primary interest for the moment. Another way of stating the second law is that the resultant force acting on a particle is proportional to the time rate of change of the momentum of the particle and that this change is in the direction of the force. The proportionality factor is the mass of the particle in question.

Both statements of Newton's second law lead to the same result in equation form:

$$\sum \vec{F} = \frac{d\vec{mv}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} \quad (2-15)$$

It will be assumed that the mass of the particle does not change during the time interval dt . Thus dmv/dt can be written mdv/dt . Also note that Σ symbol was used. The resultant acceleration of the particle is equal to the resultant vector sum of the forces acting on the particle. Thus many different forces acting on an object can be vectorially resolved into one force (see Figure 2-6).

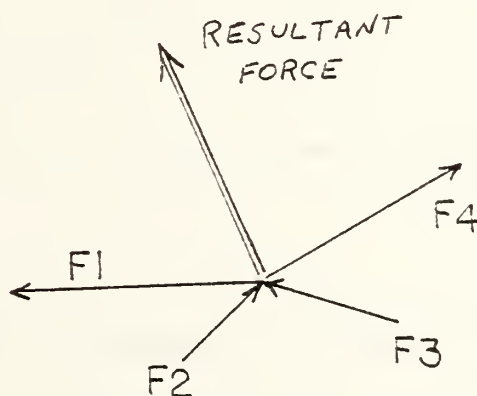


Figure 2-6. Vector Summation

The particle will respond in the same way no matter if we deal with the several individual forces or with their resultant, but the picture is greatly simplified when dealing with only a single resultant force.

Both the vectorial nature of forces and the capability to sum individual forces can be used with the axis systems previously described for the helicopter. Being a complex machine, the helicopter can have many different forces from different sources acting at various points on the body of the aircraft. These forces can be broken down into components along the aircraft axes and summed such that there now exists three mutually perpendicular forces acting at the center of gravity of the aircraft. The problem of the helicopter's reaction to these various forces is now greatly simplified (see Figure 2-7).

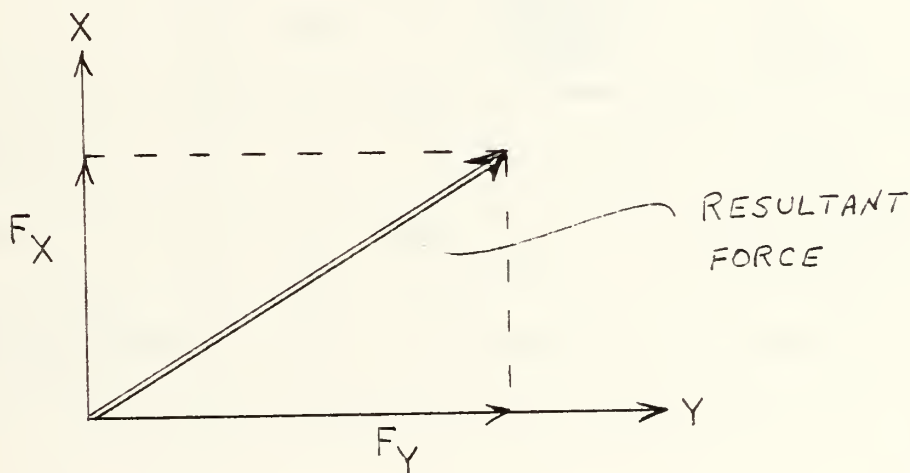


Figure 2-7. Components of a Vector in a Plane

Equation (2-15) can be written for each axis direction:

$$\sum \bar{F}_X = m\bar{a}_X ; \quad \sum \bar{F}_Y = m\bar{a}_Y ; \quad \sum \bar{F}_Z = m\bar{a}_Z \quad (2-16)$$

As an example, a single force not lying in any single plane would have components in the X, Y, and Z directions. The components of this force would be determined and would be respectively included in each of the three force equations above. If a force acts entirely along a single axis, the Z-axis for example, it would have no components in the other two directions, X or Y in this case.

Units of measure for force are listed below in Table 2-1.

Table 2-1. Units of Measure for Force

	SYMBOL	ENGLISH ENGINEERING SYSTEM	SI SYSTEM
FORCE	$\bar{F}$	Pounds Force	Newtons
MASS	m	Slugs	Kilograms
ACCELERATION	$\bar{a}$	ft/sec ²	m/sec ²

Since the consideration of variable masses will not be considered (the mass of a helicopter will not change over the time periods under consideration here) it can be seen that the acceleration of a body is directly proportional to the force acting on the body.

2. Moments

Forces applied to a particle create only linear motion of the particle. This is so because a particle is considered to be a very small quantity and any forces applied must necessarily act at its center of mass. The phenomenon of angular acceleration occurring simultaneously with linear acceleration arises when a force is applied to an object (something larger than a particle). If the direction vector of a force applied to an object does not pass directly through the object's center of mass then the object will start to rotate about its center of mass because a moment has been created.

A moment is a force applied at a distance from an axis around which an object rotates. If the object is located in free-space, it will rotate about its center of mass. If the object is pinned somehow, the pin will act as the pivot point about which the object will rotate.

In equation form, for a fixed-axis system:

$$\bar{M} = \bar{F}\bar{r} \quad (2-17)$$

where $\bar{M}$ is the moment in foot-pounds, $\bar{F}$ is the force, and $\bar{r}$ is the perpendicular distance from the object's center of mass to the force vector (see Figure 2-8).

Just like a force, a moment is a vectorial quantity but it is a rotational vector in this case. As with angular velocity, a positive moment is defined in the counterclockwise direction.

In the evaluation of the effects external moments have on the motion of the helicopter, the moment of momentum, H , is considered.

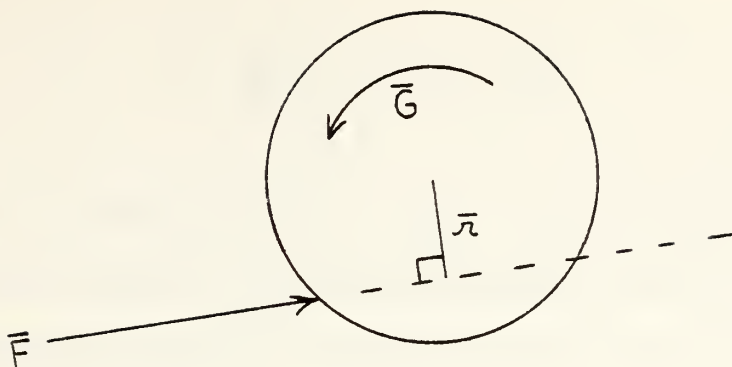


Figure 2-8. Generation of a Moment

By definition, the moment of momentum of a small part on the aircraft, dm , is:

$$\delta h = \bar{r} \times \bar{V} \, dm \quad (2-18)$$

Taking the derivative of the above yields

$$\frac{d}{dt} \delta h = \frac{d}{dt} (\bar{r} \times \bar{V}) \, dm = \frac{d\bar{r}}{dt} \times \bar{V} \, dm + \bar{r} \times \frac{d\bar{V}}{dt} \, dm \quad (2-19)$$

Considering a small chunk of mass on a rotating body, the velocity of this piece of mass is:

$$\bar{V} = \bar{V}_c + \frac{d\bar{r}}{dt} \quad (2-20)$$

Where $d\bar{r}/dt$ is the rotational velocity of the body and $\bar{V}_c$ is the velocity of the object's center of mass.

Also,

$$\bar{\mathbf{r}} \times \frac{d\bar{\mathbf{V}}}{dt} \delta m = \bar{\mathbf{r}} \times \delta \bar{\mathbf{F}} = \delta \bar{\mathbf{G}} \quad (2-21)$$

This is true since $d\bar{\mathbf{V}}/dt \delta m = \delta \bar{\mathbf{F}}$ is just $\bar{\mathbf{F}} = m\bar{\mathbf{a}}$, while $\delta \bar{\mathbf{G}}$ is the moment about the center of mass of the object produced by force $\delta \bar{\mathbf{F}}$.

Substituting equations (2-20) and (2-21) into equation (2-19) gives:

$$\delta \bar{\mathbf{G}} = \frac{d}{dt} (\delta \bar{\mathbf{h}}) - (\bar{\mathbf{V}} - \bar{\mathbf{V}}_c) \times \bar{\mathbf{V}} \delta m \quad (2-22)$$

where $\bar{\mathbf{G}}$ is the linear momentum of a particle, $m\bar{\mathbf{v}}$.

Since $\bar{\mathbf{V}} \times \bar{\mathbf{V}} = 0$, this becomes

$$\delta \bar{\mathbf{G}} = \frac{d}{dt} (\delta \bar{\mathbf{h}}) + \bar{\mathbf{V}}_c \times \bar{\mathbf{V}} dm \quad (2-23)$$

If equation (2-23) is summed for all small mass elements on the helicopter the resultant mass is the total mass of the helicopter and the resultant velocity is $\bar{\mathbf{V}}_c$. As shown before, $\bar{\mathbf{V}} \times \bar{\mathbf{V}} = 0$ and equation (2-23) reduces to

$$\bar{\mathbf{G}} = \frac{d\bar{\mathbf{h}}}{dt} \quad (2-24)$$

This equation states that the angular momentum of an object is changed when a moment is applied to the object.

D. CORIOLIS FORCES AND MOMENTS

Coriolis forces arise from the acceleration produced when a particle moves along a path in a plane which is itself rotating. A flapping blade is subject to a coriolis acceleration in the plane of rotation.

When a rotor blade flaps, its moment of inertia about the rotational axis changes. This can be seen by considering the center of mass of the blade being rotated to an increased flapping angle, $\beta + \Delta\beta$. As the blade flaps up, the center of mass moves closer to the rotational axis (see Figure 2-9).

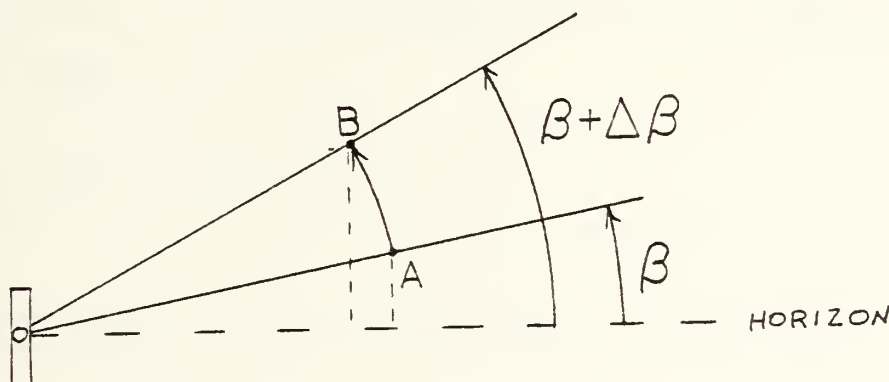


Figure 2-9. Motion of Flapping Blade

From the law of the conservation of angular momentum, the blade will experience an accelerating force if the center of mass of the blade moves closer to the rotational axis (flaps up from A to B) or a retarding force if the center of mass moves farther away (flaps down from B to

A). These forces are manifested as vibrations at the blade root, and for every harmonic of flapping there is an appropriate inplane Coriolis vibration.

To find the coriolis force the following steps are used [Ref. 3]. Point B has two components of velocity which are of interest, dr/dt along the blade projection (inwards) and $\omega r \cos (\beta + \Delta\beta)$ (in the plane of rotation). Taking components of velocity on the line perpendicular to the blade at P_2 , (see Figure 2-10) results in:

$$\frac{dr}{dt} \Delta\theta + \Omega r \cos (\beta + \Delta\beta) \quad (2-24)$$

and the difference in velocity perpendicular to the original projection between P_2 and P_1 is:

$$\frac{dr}{dt} \Delta\theta + \Omega r (\cos \beta + \Delta\beta) - \Omega r \cos \beta \quad (2-25)$$

$$= \left\{ \frac{d}{dt} (r \cos \beta) \right\} \Delta\theta + \Omega r (\cos \beta \cos \Delta\beta - \sin \beta \sin \Delta\beta) - \Omega r \cos \beta$$

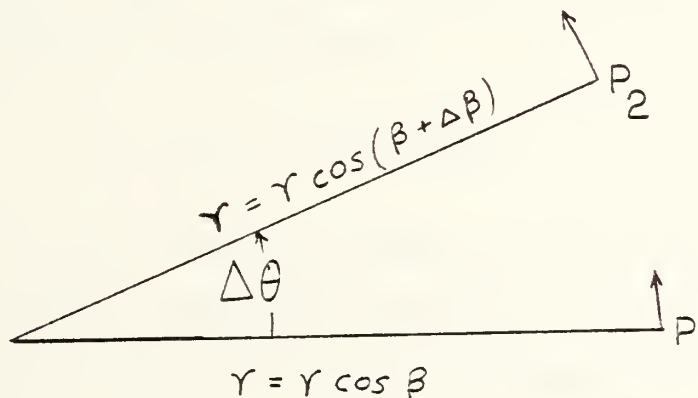


Figure 2-10. Flapping Blade in Plane of Rotation

Equation (2-25) was arrived at by using the trigonometric identity

$$\cos \beta + \Delta\beta = \cos \beta \cos \Delta\beta - \sin \beta \sin \Delta\beta \quad (2-26)$$

Using small angle theory $\cos \beta \approx 1$ and $\sin \beta \approx \beta$. This will result in

$$\Delta V = r \sin \beta \frac{d\beta}{dt} \Delta\theta - \Omega r \beta \Delta\beta \quad (2-27)$$

Dividing through by Δt to find the acceleration yields:

$$\frac{\Delta V}{\Delta t} = - r\beta \frac{d\beta}{dt} \frac{\Delta\theta}{\Delta t} - \Omega r \beta \frac{\Delta\beta}{\Delta t} \quad (2-28)$$

and since $\frac{\Delta\theta}{\Delta t} = \Omega$ and $\frac{\Delta\beta}{\Delta t} = \dot{\beta}$,

$$a_{\text{cor}} = - r\beta \dot{\beta}\Omega - \Omega r \beta \dot{\beta} \quad (2-29)$$

which gives the final form of the Coriolis acceleration:

$$a_{\text{cor}} = - 2 \Omega r \beta \dot{\beta} \quad (2-30)$$

If the Coriolis force is desired, simply multiply the mass of the blade by the acceleration obtained above.

$$F_{\text{cor}} = - 2 M_b \Omega r \beta \dot{\beta} \quad (2-31)$$

This Coriolis force can be considered to act at the center of the blade and it produces a moment about the hub of the rotor. The moment produced therefore is the force times the distance through which the force acts,

$$M_{\text{cor}} = 2 M_b r^2 \Omega \dot{\beta} \quad (2-32)$$

A useful relation can be used to shorten this equation. The moment of inertia of an object is defined as

$$I = \int y^2 dm \quad (2-33)$$

where y is the distance from an axis and dm is an elementary particle of mass. If the total mass of the object and its center of mass location is known, the moment of inertia can be expressed as

$$I = k^2 m \quad (2-34)$$

where k is the distance from the axis to the center of mass and m is the total mass. Thus, substituting r for k in equation (2-34) and substituting equation (2-34) into equation (2-32) yields

$$M_{\text{cor}} = 2 I \Omega \dot{\beta} \quad (2-35)$$

NOTE: The Coriolis force just derived is not the only Coriolis force that arises from the dynamic notions of the helicopter. It is perhaps the most significant and easily understood Coriolis force, but it must be pointed out that many Coriolis forces must be accounted for when a very rigorous analysis of the helicopter's motions is conducted. Other Coriolis forces will arise from aeroelastic effects of the rotor blades, differences in motion between the helicopter rotor and fuselage and also from fuselage aeroelastic effects. Many of these latter Coriolis forces will be very small and will not be apparent when considering the helicopter as a rigid body.

E. RIGID BODY EQUATIONS

Now that some of the background concerning the nature of forces and moments has been presented, it remains to put these ideas to use for the solution of problems dealing with helicopter stability. Simply put, the various forces and moments that act on the helicopter are resolved into components which act along or about one of the principal axes of the helicopter. The various forces and moments to be considered are the factors which give rise to the vehicle's motion.

According to Reference 4, certain assumptions concerning the helicopter's motions and references will have to be made in order to establish the ground rules for further analysis and also to simplify the systems of equations that will arise.

Assumption 1: The helicopter is a rigid body.

A rigid body is one in which motions between individual mass elements that make up the body do not occur. In this way, distances between specified points in the helicopter's fuselage remain fixed. Thus no bending or twisting of the fuselage is considered as the helicopter moves through space.

This assumption allows the helicopter's motion through space to be described as the rectilinear motion of the center of mass of the aircraft and by the curvilinear motion of the same point. In reality, the fuselage does bend and twist to a certain degree in flight. The dynamics of the structure are changed when these aeroelastic effects are considered. The aeroelastic effects serve to greatly increase the degrees of freedom that must be considered when analyzing the equations

of motion. A rigid body will be assumed here since aeroelasticity is beyond the purpose of this paper and consequently the solution to generated problems will not be overly complicated.

Assumption 2: The earth is considered to be flat and is fixed in space.

This assumption is made to negate any minor corrections that may otherwise have to be made for the gravity constant or for a moving inertial reference frame. Negligible error is introduced by this assumption since the altitudes, air speeds, and time lengths under consideration are small.

Assumption 3: The helicopter is assumed to be in a trimmed level flight condition, or in a hover. Small perturbations in the helicopter's motion are then considered.

This assumption allows the linearizing of normally nonlinear responses by considering only small increments of motion. Simplification of the required equations used to solve for the helicopter's motion is the result. While the analyses will be performed for very small changes in motion, the results can be extended to larger motions without very much loss in accuracy.

Other mathematical benefits arising from this assumption include the small angle approximations for sine and cosine:

$$(1) \sin \alpha = \alpha$$

$$(2) \cos \alpha = 1$$

where α is measured in radians, and the fact that the power of any small increment can be ignored when compared to the original: $\varepsilon^2 \approx 0$ when ε is small.

Assumption 4: The helicopter is trimmed in steady, level flight and the longitudinal forces and moments due to lateral perturbations from trim are negligible.

This assumption discounts any initial angular velocities in roll or yaw (P_0 , R_0) and any initial lateral velocity (V_0). Furthermore, any lateral perturbations that do arise will not affect the longitudinal response of the aircraft. This last statement is important since the longitudinal and lateral equations of motion are thereby decoupled.

Assumption 5: The X-Z plane is a plane of symmetry.

This is usually the case for most flight vehicles. Although this is not true for many helicopters, very little error is introduced into the final equations of motion when this assumption is made. With symmetry and with Assumption 4 considered all rolling moments, yawing moments, and side forces are reduced to zero. Consequently the longitudinal equations of motion can be described by just three equations: those dealing with longitudinal and vertical forces and with the pitching moment.

Combining these five assumptions makes it possible to describe the helicopter's motion with just six equations. This is so because the assumptions allowed many additional degrees of freedom to be dropped from consideration. Consequently, three force equations and three moment equations are used to describe the helicopter's motion.

Note in particular Assumption 4. Since the longitudinal and lateral-directional motions are now decoupled, only those forces and moments which affect the longitudinal motions of the helicopter need be considered. These turn out to be the X and Z forces and the M moment.

1. Three Force Equations

Since forces act in straight lines, it is very convenient to resolve the components of given forces acting on the helicopter along the X, Y, and Z axes. Only external forces on the aircraft will be considered. Internal forces must necessarily be opposed by other internal forces such that the sum total of all the internal forces on the helicopter's fuselage is zero. While internal forces may play an important part in airframe structural considerations, they play no part in the analysis of the helicopter's motions. The external forces that will be considered arise from three principal sources: gravity forces, inertia forces, and aerodynamic forces.

2. Three Moment Equations

The same principles that were used for the determination of the three force equations are also used for moments. Here, however, instead of the X, Y, and Z axes acting as directions along which forces are measured, the axes act as lines around which the moments turn. Nomenclature of the moments will be briefly reviewed:

L-Moment: Rotates about the X-axis; positive is right wing down.

M-Moment: Rotates about the Y-axis; positive is nose up.

N-Moment: Rotates about the Z-axis; positive is nose right.

F. EXPANSION OF FORCES AND MOMENTS

The equations for the generalized aerodynamic forces and moments acting on an aircraft have commonly been written in the form

$$A = C_A \frac{1}{2} \rho V^2 S \quad (2-36)$$

where A stands for any desired force or moment, C_A is a nondimensional coefficient, ρ is air density, V is the steady-state velocity of the vehicle, and S is the surface area. It can be seen by this equation that the aerodynamic forces and moments generated on the helicopter are dependent upon the density of the air through which the vehicle is flying and also the velocity of the aircraft relative to the air.

The nondimensional coefficients, C_A , are also dependent on Reynolds and Mach numbers, angles of attack and sideslip, and linear and angular velocity and their derivatives. If the aerodynamic forces are considered to be continuous functions of all these variables, each of the forces and moments (X , Y , Z and L , M , N respectively) can be expressed in terms of the variables by expanding the terms in a Taylor series [Ref. 4].

A Taylor series for the effect of forward velocity changes has the form:

$$F = F_0 + \frac{\partial F}{\partial u} \Delta u + \frac{\partial^2 F}{\partial u^2} \frac{\Delta u^2}{2!} + \frac{\partial^3 F}{\partial u^3} \frac{\Delta u^3}{3!} + \dots \quad (2-37)$$

All partial derivative terms starting with the second derivative are higher-order terms and can be neglected by means of Assumption 3 in the previous section without changing the accuracy of the solution. This will help to simplify the equation by eliminating many terms which serve to complicate the problem but contribute very little to the value of the final solution.

The force expression is now reduced to one showing the initial trim condition and all changes to the force resulting from the first partial derivative of force with respect to major variables and their derivatives when applicable.

$$F = F_0 + \frac{\partial F}{\partial u} \Delta u + \frac{\partial F}{\partial z} \Delta z + \frac{\partial F}{\partial \theta} \Delta \theta + \frac{\partial F}{\partial \dot{u}} \Delta \dot{u} + \frac{\partial F}{\partial \dot{z}} \Delta \dot{z} + \dots \quad (2-38)$$

Since the change in force from a steady flight condition due to perturbations is desired and not the total force, the trim force can be subtracted from both sides of the equation. The result is:

$$\Delta F = \frac{\partial F}{\partial u} \Delta u + \frac{\partial F}{\partial z} \Delta z + \frac{\partial F}{\partial \theta} \Delta \theta + \frac{\partial F}{\partial \dot{u}} \Delta \dot{u} + \frac{\partial F}{\partial \dot{z}} \Delta \dot{z} + \dots \quad (2-39)$$

III. CALCULATION OF THE STABILITY DERIVATIVES

Probably the most limiting factor of expanding the helicopter's utility to a multi-mission role has been its relatively slow cruise speed. Therefore much emphasis has been directed in recent years towards increasing the maximum forward flight velocity of helicopters. The stability response of helicopters in hover and in slow flight can generally be classified as one consisting of two heavily damped roots and a divergent oscillatory motion. These characteristics can (and usually do) change as the aircraft increases its forward speed.

Even though the response of a helicopter in a hover is generally known, its response at high forward flight velocities cannot accurately be predicted. Therefore, the engineer must be able to calculate the stability derivatives for the aircraft in question and at the proper flight condition (altitude and airspeed) so that he may solve for the helicopter's modes of response. The equations and procedures presented in this chapter outline how the calculation of those stability derivatives are carried out. The charts used for this procedure apply to the regime of high forward flight velocity with an advance ratio of 0.3 or greater.

A. TYPES AND USES

Both dimensional and non-dimensional stability derivatives are used for the solution of problems dealing with aircraft stability and control.

Each type derivative has certain useful properties. By using non-dimensional derivatives, the stability characteristics of aircraft can be compared regardless of size.

Non-dimensional derivatives are concerned with force and moment coefficients and with non-dimensional velocity and time. The real advantage to using non-dimensional stability derivatives occurs when comparing the stability values between different sizes of the same aircraft, for example comparing data obtained from a one-tenth scale model in a wind tunnel with data expected or obtained from flight tests of the full scale aircraft. This capability of non-dimensional derivatives to allow the comparison of data in this manner makes possible the prediction of stability characteristics of aircraft based on tests performed on scale models. It is immediately apparent that this is an economical benefit when the alternative is to perform all tests and development on full scale aircraft.

Dimensional derivatives are good for measuring direct forces and moments of the aircraft. The use of dimensional stability derivatives lead directly to numerical coefficients in the sets of simultaneous differential equations describing the real time dynamics of the airframe. By analyzing the dynamic response numbers thus obtained, the helicopter's stability characteristics can be ascertained. Once this is known, the amount of control necessary to reach the desired flying qualities can be added.

Care should be taken when comparing the values of stability derivatives received from different sources. It is important to note that

both dimensional and non-dimensional derivatives are widely used throughout the industry and that different methods may be used to non-dimensionalize or normalize the derivatives for a particular aircraft.

Thus if the values of the stability derivatives for a given aircraft from one source are found to differ from those values received from another source, the most likely cause for the difference is that a different system of non-dimensionalizing or normalizing the derivatives has been used by each source.

It should be noted that one common method used for dimensional derivatives is to normalize the force derivatives by the mass of the aircraft and the moment derivatives by the aircraft's moment of inertia about the pitching axis. This method is used here.

In this chapter the procedures will be outlined for calculating the stability derivatives X_u , X_w , Z_u , Z_w , M_u , M_w , and M_q . These derivatives are needed to solve the stability determinant which will yield the modes of motion of the helicopter. These procedures are presented in more detail in [Ref. 5].

Initially, certain geometric data must be known about the helicopter. The forward flight velocity at which the stability condition will be evaluated must also be chosen, and the trim condition of the helicopter computed. Since the aerodynamic forces which act on the helicopter are dependent on flight velocity, the trim condition and subsequent stability derivatives will be different for each velocity evaluated.

After the trim conditions of the helicopter are found at the desired airspeed, the isolated derivatives are determined and corrected for main rotor solidity to yield the local derivatives. From these, the total derivatives may be calculated.

B. DEFINITION OF TERMS

The following definitions will be used:

a	=	Lift curve slope of the rotor blade
a_0	=	Blade coning angle (radians)
a_1	=	Longitudinal flapping angle (radians)
AR	=	Aspect ratio
$A_{X_{FUS}}$	=	Fuselage frontal area (sq ft)
$A_{Z_{FUS}}$	=	Fuselage planform area (sq ft)
b	=	Number of rotor blades
b_1	=	Lateral flapping angle (radians)
c	=	Blade chord (ft)
C_{D_0}	=	Profile drag coefficient
C_D	=	Drag coefficient = $D/\frac{1}{2}\rho V^2 S$
C_D'	=	Drag coefficient of the main rotor = $D/T.F.$
C_L	=	Lift coefficient = $L/\frac{1}{2}\rho V^2 S$
C_L'	=	Lift coefficient of the main rotor = $L/T.F.$
C_Q	=	Rotor torque coefficient = $Q/T.F. R$
D	=	Aerodynamic drag force (lbs)
e	=	Blade hinge offset
I_b	=	Blade moment of inertia about flapping hinge (slug-sq ft)
I_Y	=	Aircraft moment of inertia about the pitching axis
K	=	Downwash interference factor

L	=	Aerodynamic lift force (lbs)
l_X	=	Longitudinal moment arm, positive when the point of application of the force vector is forward from the C.G. position (ft)
l_Z	=	Normal (vertical) moment arm, positive when the point of application of the force vector is below the C.G. position (ft)
M	=	Pitching moment of an aircraft component (ft-lb)
M_u, M_w, M_q	=	Pitching moment total derivatives
M_S	=	First moment of blade mass about the flapping hinge (slug-ft)
M_T	=	Mach number of advancing blade tip
Q	=	Rotor torque (ft-lbs)
q_0	=	Dynamic pressure = $\frac{1}{2}\rho V^2$ (lb/sq ft)
R	=	Rotor radius (ft)
S	=	Area of an aerodynamic surface (sq ft)
T	=	Rotor thrust (lbs)
$T.F.$	=	Thrust factor = $\rho\pi R^2(\Omega R)^2$ (lbs)
V_0	=	Steady state or trim value of velocity (ft/sec)
V_S	=	Velocity of sound in standard atmosphere (ft/sec)
W	=	Aircraft gross weight (lbs)
X	=	Longitudinal force along the body X-axis (lbs)
X_u, X_w	=	Total stability derivatives of the longitudinal X-force
Z	=	Normal force along the body Z-axis (lbs)
Z_u, Z_w	=	Total stability derivatives of the normal Z-force
α	=	Remote wind angle of attack relative to body X-axis (radians)
α_C	=	Rotor angle of attack (radians)
γ	=	Lock Intertia number = $\frac{\rho a c R^4}{I_b}$
ε	=	Downwash interference angle (radians)

- λ = Rotor inflow ratio = $\frac{V_o \sin \alpha_c - V_i}{\Omega R}$
 μ = Rotor tip speed ratio = $\frac{V_o \cos \alpha_c}{\Omega R}$
 ρ = Air density (slug/cu ft)
 σ = Rotor solidity = $\frac{b c}{\pi R}$
 Ω = Rotor angular velocity (radians/second)
 θ = Blade twist (radians)
 $\theta_{.75}$ = Blade section pitch angle at .75 radius (radians)

SUBSCRIPTS

- F = Pertaining to the front rotor
 FUS = Pertaining to the fuselage
 T = Pertaining to the horizontal tailplane
 TR = Pertaining to the tail rotor.

The following procedures give a step-by-step approach that can be followed to find the desired total stability derivatives. The calculation of the stability derivatives is nothing more than an accounting of the forces and moments acting on the aircraft. Because of the number of calculations involved and the interaction of many different components, however, the calculation procedure is necessarily a lengthy one.

C. HELICOPTER DATA

Information about the physical layout of the helicopter must be available before the computations can begin. The following must be known:

$$a, b, c, l, R, \Omega, W, I_b, I_Y, M_S, A_X, A_Z$$

$$(C_{D_o})_T, (AR)_T, S_T, a_T$$

Because of the high aspect ratio of a helicopter blade, the lift curve slope of the blade, a , can generally be taken as 5.73 per radian. Establish the altitude and flight speed, V_0 , at which the derivatives are desired to be known. The altitude will determine the atmospheric density. For sea level, $\rho = .002377$ slugs/cu ft.

Knowing the above, the following parameters can be calculated:

$$\sigma, \mu, M_T, V_S, q_0, T.F., \text{Rotor Tip Speed}$$

D. CALCULATION OF HELICOPTER TRIM VALUES

First approximations of the fuselage lift and drag coefficients are determined with the assumption that $\alpha_{FUS} = 0$. C_L and C_D will be recalculated when more precise information is known about α_{FUS} . A theoretical or experimentally obtained graph of $C_{L_{FUS}}$, $C_{D_{FUS}}$ and $C_{M_{FUS}}$ versus α_{FUS} is needed to begin the computation. Figure 10.1-3 [Ref. 5] is an example of such a graph.

The first approximation of fuselage lift and drag will be made using the coefficients just obtained.

$$L_{FUS} = C_{L_{FUS}} q_0 A_{Z_{FUS}} \quad (3-1)$$

$$D_{FUS} = C_{D_{FUS}} q_0 A_{X_{FUS}} \quad (3-2)$$

The initial estimate of main rotor lift and drag forces can now be made.

$$L = W - L_{FUS} \quad (3-3)$$

$$D = -D_{FUS} \quad (3-4)$$

The rotor lift and drag coefficients are non-dimensional and are obtained by dividing main rotor lift by the thrust factor. Both are normalized by rotor solidity.

$$\left(\frac{C_L'}{\sigma} \right) = \frac{L}{T.F.\sigma} \quad (3-5)$$

$$\left(\frac{C_D'}{\sigma} \right) = \frac{D}{T.F.\sigma} \quad (3-6)$$

Reference 6 contains theoretical rotor data performance charts for hinged rectangular planform blades of various degrees of twist. Different charts are used for the variables of blade twist, tip speed ratio and advancing tip mach number. All charts are based on a rotor solidity of 0.1. Corrections must be made if the rotor solidity of the helicopter under evaluation varies from 0.1. The charts are entered knowing:

$\frac{C_D'}{\sigma}$ and $\frac{C_L'}{\sigma}$ to find $\frac{C_0}{\sigma}$, a_1 , α_c and $\theta.75$.

$$\Delta\sigma = \sigma - 0.1 \quad (3-7)$$

$$\left(\frac{C_L'}{\sigma} \right)_{0.1} = \frac{C_L'}{\sigma} \quad (3-8)$$

$$\left(\frac{C_D'}{\sigma} \right)_{0.1} = \frac{C_D'}{\sigma} - \frac{\Delta\sigma}{2\mu^2} \left(\frac{C_L'}{\sigma} \right)^2 \quad (3-9)$$

The above two quantities, $\left(\frac{C_L'}{\sigma}\right)_{0.1}$ and $\left(\frac{C_D'}{\sigma}\right)_{.1}$, together with the known or calculated quantities blade twist angle, θ , main rotor tip speed ratio, μ , and mach number of the advancing blade tip, M_T , are needed to use the charts in [Ref. 5]. These charts will yield an approximation of the following main rotor trim parameters:

- (1) α_c Rotor angle of attack
- (2) a_0 Blade coning angle
- (3) a_1 Longitudinal flapping angle
- (4) b_1 Lateral flapping angle
- (5) $\theta_{.75}$ Blade section pitch angle at .75 radians
- (6) λ Inflow ratio
- (7) $\frac{C_Q}{\sigma}$ Coefficient of rotor torque = $\frac{Q}{T.F.R}$

The angle of attack of the main rotor, α_c , and the rotor torque, Q , may now be calculated in the following manner:

$$\alpha_c = (\alpha_c)_{0.1} + \frac{\Delta\sigma}{2\mu^2} \left(\frac{C_L'}{\sigma}\right) \quad (3-10)$$

$$Q = \frac{C_Q}{\sigma} (T.F.) \sigma R \quad (3-11)$$

Interference effects can exist when the downwash created by one aerodynamic component affects the performance of another aerodynamic component. These effects can be described as changes in the local angle of attack and the local velocity. Changes in the local velocity are

usually small, however, and will not be considered here. The local angle of attack can be expressed by the following equation:

$$\alpha_{\text{LOCAL}} = \alpha + i - \varepsilon \quad (3-12)$$

where α is the remote wind angle of attack with respect to the X-axis

i is the geometric inclination of the aerodynamic component being evaluated with respect to the X-axis

ε is the aerodynamic interference angle.

For helicopters, aerodynamic interference is produced mainly by the main rotor downwash. The downwash velocity of a rotor varies with time as well as position and the result is an exceedingly complicated situation to evaluate. Fortunately, measurements of lift and drag for a single rotor helicopter as reported in Reference 5 show that interference effects between the main rotor and fuselage and the main rotor and horizontal tail are negligible.

The interference between the front and rear rotors of a tandem rotor configuration is more significant. Tandem rotors will not be evaluated here, however, Reference 5 contains charts and procedures for calculating this interference effect.

The downwash interference angles, ε , are usually small. They can be calculated using equation (3-13).

$$\varepsilon_{\text{FUS}} = \varepsilon_{\text{T}} = \varepsilon_{\text{TR}} = K \left(\tan \alpha_{\text{c}} - \frac{\lambda}{\mu} \right) \quad (3-13)$$

where K is the rotor interference factor and is usually equal to 1.0 for single rotor aircraft.

The above calculations have revealed additional information about the trim condition of the helicopter. A second approximation of the fuselage trim angle of attack can now be made. First a mathematical relationship is obtained between α_{FUS} and $C_{M_{FUS}}$. This information is then plotted against experimental data for the same two factors. The intersection of the two curves will yield the trim value for α_{FUS} .

Use equation (3-14) to find the relation between α_{FUS} and $C_{M_{FUS}}$.

$$\alpha_{FUS} = \frac{[l_X L - l_Z D + \frac{eb\Omega^2 M_s}{2} (a_1 + \alpha_C - i)]_F + q_0 [l_X S a (i - \varepsilon)]}{- (l_X D) + l_Z L)_F - q_0 (l_X S a)_T} + \frac{C_{M_{FUS}} q_0 A_{X_{FUS}} l_{FUS}}{- (l_X D) + l_Z L)_F - q_0 (l_X S a)_T} - \varepsilon_{FUS} \quad (3-14)$$

The only two unknowns in equation (3-14) are α_{FUS} and $C_{M_{FUS}}$. The result will be that some constant times $C_{M_{FUS}}$ will equal α_{FUS} . Plot this against the experimental data for the values such as shown in Figure 10.1-11 [Ref. 5]. The intersection of the two lines will be the new approximation of α_{FUS} .

The recently obtained value of α_{FUS} can be used to gain a second approximation of $C_{L_{FUS}}$, $C_{D_{FUS}}$, $C_{N_{FUS}}$, and $C_{M_{FUS}}$ by using the M_{FUS} vs. α_{FUS} figure again.

A refined approximation of the fuselage forces can be gained by using equations (3-1) and (3-2). The fuselage pitching moment can be calculated from equation (3-15).

$$M_{FUS} = C_{M_{FUS}} q_0 A_{X_{FUS}} l_{FUS} \quad (3-15)$$

Tail Rotor Calculations

The yawing moment of the fuselage is calculated next.

$$N_{FUS} = C_{N_{FUS}} q_0 A_{X_{FUS}} l_{FUS} \quad (3-16)$$

The tail rotor thrust must be equal to the yaw moments produced by the main rotor and the fuselage. Thus:

$$T_{TR} = \frac{N_{FUS} + Q_F}{-l_{X_{TR}}} \quad (3-17)$$

The coefficient of tail rotor thrust can now be obtained.

$$\left(\frac{C_L'}{\sigma}\right)_{TR} = \left[\frac{T}{(T.F.)\sigma}\right]_{TR} \quad (3-18)$$

Knowing the values of θ , TR , μ_{TR} , and $M_{T_{TR}}$ (calculated previously) plus $\frac{C_L'}{\sigma}_{TR}$ and assuming $(\alpha_c)_{TR} = 0$, use the appropriate charts in [Ref. 5]

to find:

$$\left[\left(\frac{C_D'}{\sigma}\right)\right]_{TR} \quad 0.1, \quad \left(\frac{C_Q'}{\sigma}\right)_{TR}, \quad \lambda_{TR}, \quad (0.75)_{TR}$$

compute:

$$\left(\frac{C_D'}{\sigma}\right)_{TR} = \left[\left(\frac{C_D'}{\sigma}\right)_{0.1} + \frac{\Delta\sigma}{2\mu^2} \left(\frac{C_L'}{\sigma}\right)^2\right]_{TR} \quad (3-19)$$

The drag of the tail rotor can now be calculated:

$$D_{TR} = \left[\frac{C_D'}{\sigma} \text{ (T.F.) } \sigma \right]_{TR} \quad (3-20)$$

The only tail rotor derivative needed is:

$$\frac{\partial D_{TR}}{\partial u_{TR}} = \left(\frac{\text{T.F. } \sigma}{\Omega R} \right)_{TR} \left(\frac{\partial \frac{C_D'}{\sigma}}{\partial \mu} \right)_{TR} \quad (3-20a)$$

The last term in equation (3-20a) is obtained from the appropriate chart of Reference 5 knowing μ , M_T , and $\left(\frac{C_L'}{\sigma} \right)_{TR}$.

The characteristics of the horizontal tail plane are determined from the following equations:

$$\alpha = \alpha_{FUS} + \epsilon_{FUS} \quad (3-21)$$

$$\alpha_T = \alpha + L_T - \epsilon_T \quad (3-22)$$

$$C_{L_T} = a_T \alpha_T \quad (3-23)$$

$$C_{D_T} = \left(C_{D_0} + \frac{C_{L_T}^2}{\pi A R} \right)_T \quad (3-24)$$

$$L_T = C_{L_T} q_0 S_T \quad (3-25)$$

$$D_T = C_{D_T} q_0 S_T \quad (3-26)$$

Forces are summed in the X and Z directions to obtain a new approximation of the main rotor lift and drag. The following equations are used:

$$K_1 = W\alpha - L_{FUS} (\alpha - \varepsilon_{FUS}) - L_T (\alpha - \varepsilon_T) + D_{FUS} + D_T + D_{TR} \quad (3-27)$$

$$K_2 = D_{FUS} (\alpha - \varepsilon_{FUS}) + D_T (\alpha - \varepsilon_T) + D_{TR} (\alpha - \varepsilon_{TR}) + L_{FUS} + L_T - W \quad (3-28)$$

where K_1 represents the total drag being developed by the helicopter and K_2 represents the total lift.

The lift of the main rotor can now be approximated from the following two equations:

$$L_F = K_1 \alpha - K_2 \quad (3-29)$$

$$D_F = \frac{L\alpha - K_1}{1 - \alpha^2} \quad (3-30)$$

The angle of attack used here is from equation (3-16).

Equations (3-5 through 3-9) are used to obtain better estimates for

$$\left(\frac{C_L'}{\sigma}\right), \left(\frac{C_L'}{\sigma}\right)_{0.1}, \left(\frac{C_D'}{\sigma}\right), \text{ and } \left(\frac{C_D'}{\sigma}\right)_{0.1}.$$

The steps following equation (3-9) to equation (3-25) should be repeated until the trim values converge. This should only take one or two iterations. The result of the above calculations will yield the final trim values of the helicopter at one airspeed. These values are needed for the further calculation of the stability derivatives.

E. CALCULATION OF ROTOR ISOLATED DERIVATIVES

Isolated rotor derivatives are defined as those aerodynamic parameters for the rotor that change with respect to tip speed ratio, μ , rotor angle of attack, α_c , and blade section pitch angle at 0.75 radius, $\theta_{.75}$. Theoretical values of these derivatives are plotted on charts and presented in Reference 5 as functions of σ , θ , γ , and μ .

These charts were derived for rotor solidity equal to 0.1. If the actual rotor solidity for the helicopter being evaluated differs from 0.1, then corrections should be made to the values obtained from these charts.

The parameters which must be corrected for solidity are those dealing with μ and α_c . The correction factors and equations to use when correcting for solidity are listed below.

Solidity Corrections for (μ) Derivatives

$$\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu} = K_1 \left\{ \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu} \right]_{0.1} + \frac{\Delta\sigma}{\mu^3} \left(\frac{C_L'}{\sigma} \right) \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_c} \right]_{0.1} \right\} \quad (3-31)$$

where

$$\Delta\sigma = \sigma - 0.1 \quad (3-32)$$

$$K_1 = \frac{1}{1 + \frac{\Delta\sigma}{2\mu^2} \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_c} \right]_{0.1}} \quad (3-33)$$

()_{0.1} - denotes stability derivatives for rotor solidity $\sigma = 0.1$. These values can be directly obtained from the charts of Section 7.5. in Reference 5.

$$\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu} = \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\mu} \right]_{0.1} + K_2 \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_c} \right]_{0.1} - \left(\frac{C_L'}{\sigma} \right) \left\{ K_2 - \frac{\Delta\sigma}{2\mu^2} \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu} \right] \right\} \quad (3-34)$$

where

$$K_2 = \frac{\Delta\sigma}{2\mu^2} \left[\frac{2}{\mu} \left(\frac{C_L'}{\sigma} \right) - \frac{\partial(\frac{C_L'}{\sigma})}{\partial\mu} \right] \quad (3-35)$$

$$\frac{\partial a_1}{\partial\mu} = \left(\frac{\partial a_1}{\partial\mu} \right)_{0.1} + K_2 \left(\frac{\partial a_1}{\partial\alpha_c} \right)_{0.1} \quad (3-36)$$

$$\frac{\partial\lambda}{\partial\mu} = \left(\frac{\partial\lambda}{\partial\mu} \right)_{0.1} + K_2 \left(\frac{\partial\lambda}{\partial\alpha_c} \right)_{0.1} \quad (3-37)$$

Solidity Corrections for (α_c) Derivatives

$$\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_c} = K_1 \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_c} \right]_{0.1} \quad (3-38)$$

$$\frac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_c} = K_1 \left\{ \left[\frac{\partial(\frac{C_D'}{\sigma})}{\partial\alpha_c} \right]_{0.1} + \frac{\Delta\sigma}{\mu^2} \left(\frac{C_L'}{\sigma} \right) \left[\frac{\partial(\frac{C_L'}{\sigma})}{\partial\alpha_c} \right]_{0.1} \right\} \quad (3-39)$$

$$\frac{\partial a_1}{\partial\alpha_c} = K_1 \left(\frac{\partial a_1}{\partial\alpha_c} \right)_{0.1} \quad (3-40)$$

$$\frac{\partial\lambda}{\partial\alpha_c} = K_1 \left(\frac{\partial\lambda}{\partial\alpha_c} \right)_{0.1} \quad (3-41)$$

Obtain the following μ derivatives from Section 7.5 of [Ref. 5]:

$$\frac{\partial \lambda}{\partial \mu}, \quad \frac{\partial a_1}{\partial \mu}, \quad \frac{\partial \left(\frac{C_L'}{\sigma} \right)}{\partial \mu}, \quad \frac{\partial \left(\frac{C_D'}{\sigma} \right)}{\partial \mu}$$

and the α_c derivatives:

$$\frac{\partial \left(\frac{C_L'}{\sigma} \right)}{\partial \alpha_c}, \quad \frac{\partial \left(\frac{C_D'}{\sigma} \right)}{\partial \alpha_c}, \quad \frac{\partial \lambda}{\partial \alpha_c}, \quad \frac{\partial a_1}{\partial \alpha_c}$$

F. CALCULATION OF THE LOCAL DERIVATIVES

Using the values obtained from the charts in Reference 5 and corrected for solidity, if necessary, calculate the following dimensional derivatives for the main rotor. These local derivatives are dimensional expressions of the change of local forces or moments of various components with respect to the local wind conditions.

$$\text{u derivatives:} \quad \frac{\partial L_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega R} \right] \left[\frac{\partial \left(\frac{C_L'}{\sigma} \right)}{\partial \mu} \right] \quad \frac{\text{lb-sec}}{\text{ft}} \quad (3-42)$$

$$\frac{\partial D_F}{\partial u_F} = \left[\frac{(T.F.)\sigma}{\Omega R} \right] \left[\frac{\partial \left(\frac{C_D'}{\sigma} \right)}{\partial \mu} \right] \quad \frac{\text{lb-sec}}{\text{ft}} \quad (3-43)$$

$$\frac{\partial a_{1F}}{\partial u_F} = \left[\frac{1}{\Omega R} \right] \left[\frac{\partial a_1}{\partial \mu} \right] \quad \frac{\text{rad-sec}}{\text{ft}} \quad (3-44)$$

$$\frac{\partial M_{HUB_F}}{\partial u_F} = \left[\frac{eb\Omega M_s}{2} \right] \left[\frac{\partial a_{1F}}{\partial u_F} \right] \quad \text{lb-sec} \quad (3-45)$$

$$\alpha_F \text{ derivatives: } \frac{\partial L_F}{\partial \alpha_F} = [(T.F.)\sigma] \left[\frac{\partial \left(\frac{C_L'}{\sigma} \right)}{\partial \alpha_C} \right] \quad \text{lb/rad} \quad (3-46)$$

$$\frac{\partial D_F}{\partial \alpha_F} = [(T.F.)\sigma] \left[\frac{\partial \left(\frac{C_D'}{\sigma} \right)}{\partial \alpha_C} \right] \quad \text{lb/rad} \quad (3-47)$$

$$\frac{\partial M_{HUB_F}}{\partial \alpha_F} = \left[\frac{eb\Omega^2 M_s}{2} \right] \left[\frac{\partial a_1}{\partial \alpha_C} \right] \quad \frac{\text{lb-ft}}{\text{rad}} \quad (3-48)$$

$$q \text{ derivatives: } \frac{\partial a_{1F}}{\partial q_F} = - \left[\frac{34}{\gamma\Omega(1.883 - \mu^2)} \right] \quad \frac{1}{\text{sec}} \quad (3-49)$$

The isolated derivatives for the fuselage are found next. Using the figure referred to in Section D and the trim values for the fuselage, determine:

$$\frac{\partial C_{L_{FUS}}}{\partial \alpha_{FUS}} \quad \frac{\partial C_{D_{FUS}}}{\partial \alpha_{FUS}} \quad \frac{\partial C_{M_{FUS}}}{\partial \alpha_{FUS}}$$

These values are used next to compute the local derivatives of the fuselage. These are dimensional derivatives.

$$u_{FUS} \text{ derivatives: } \frac{\partial L_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} L_{FUS} \quad \frac{\text{lb-sec}}{\text{ft}} \quad (3-50)$$

$$\frac{\partial D_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} D_{FUS} \quad \frac{\text{lb-sec}}{\text{ft}} \quad (3-51)$$

$$\frac{\partial M_{FUS}}{\partial u_{FUS}} = \frac{2}{V_0} M_{FUS} \quad \frac{\text{lb-sec}}{\text{ft}} \quad (3-52)$$

$$\alpha_{FUS} \text{ derivatives: } \frac{\partial L_{FUS}}{\partial \alpha_{FUS}} = q_0 A_{Z_{FUS}} \left(\frac{\partial C_{L_{FUS}}}{\partial \alpha_{FUS}} \right) \quad \frac{lb}{rad} \quad (3-53)$$

$$\frac{\partial D_{FUS}}{\partial \alpha_{FUS}} = q_0 A_{X_{FUS}} \left(\frac{\partial C_{D_{FUS}}}{\partial \alpha_{FUS}} \right) \quad \frac{lb}{rad} \quad (3-54)$$

$$\frac{\partial M_{FUS}}{\partial \alpha_{FUS}} = q_0 A_{X_{FUS}} \left(\frac{\partial C_{M_{FUS}}}{\partial \alpha_{FUS}} \right) \quad \frac{lb-ft}{rad} \quad (3-55)$$

The derivatives for the horizontal tail are obtained using the charts and procedures in Reference 7. Determination of the local derivatives for the horizontal tail can be made using the following equations:

$$u \text{ derivatives: } \frac{\partial L_T}{\partial u_T} = \frac{2}{V_0} L_T \quad \frac{lb-sec}{ft} \quad (3-56)$$

$$\frac{\partial D_T}{\partial u_T} = \frac{2}{V_0} D_T \quad \frac{lb-sec}{ft} \quad (3-57)$$

$$\alpha \text{ derivatives: } \frac{\partial L_T}{\partial \alpha_T} = q_0 a_T S_T \quad \frac{lb}{rad} \quad (3-58)$$

$$\frac{\partial D_T}{\partial \alpha_T} = \frac{2 L_T}{\pi (AR)_T} a_T \quad \frac{lb}{rad} \quad (3-59)$$

G. CALCULATION OF THE TOTAL STABILITY DERIVATIVES

The total stability derivatives can finally be determined. The calculations are based on the isolated and local derivatives found in the previous sections.

Some angular derivatives needed for the calculation of the total stability derivatives are as follows:

$$\frac{\partial \alpha_{FUS}}{\partial \alpha} = \frac{\partial \alpha_T}{\partial \alpha} = 1 - K_{FT} \left(1 - \frac{1}{\mu} \frac{\partial \lambda}{\partial \alpha_c} \right) \quad (3-60)$$

$$\frac{\partial \alpha_{FUS}}{\partial u} = \frac{\partial \alpha_T}{\partial u} = - \frac{K_{FT}}{V_0} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu} \right) \quad (3-61)$$

$$\frac{\partial \alpha}{\partial u} = - K_{RF} \left(\frac{\lambda}{\mu} - \frac{\partial \lambda}{\partial \mu} \right)_R \quad (3-62)$$

$$\frac{\partial \alpha_F}{\partial \alpha} = 1 - K_{RF} \left(1 - \frac{1}{\mu} \frac{\partial \lambda}{\partial \alpha_c} \right)_R \quad (3-63)$$

The X-Force Derivatives

$$(a) \quad X_u (X_u)_F + (X_u)_{FUS} + (X_u)_T \quad (3-64)$$

where

$$(X_u)_F = \frac{\partial L_F}{\partial u_F} \alpha - \frac{\partial D_F}{\partial u_F} \quad (3-65)$$

$$\begin{aligned} (X_u)_{FUS} &= \frac{\partial L_{FUS}}{\partial u_{FUS}} (\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial u_{FUS}} \\ &+ \frac{\partial \alpha_{FUS}}{\partial u} \left[\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} (\alpha - \epsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial \alpha_{FUS}} \right] \end{aligned} \quad (3-66)$$

$$(X_u)_T = \frac{\partial L_T}{\partial u_T} (\alpha - \epsilon_T) - \frac{\partial D_T}{\partial u_T} \frac{\partial \alpha_T}{\partial u} \left[\frac{\partial L_T}{\partial \alpha_T} (\alpha - \epsilon_T) - \frac{\partial D_T}{\partial \alpha_T} \right] \quad (3-67)$$

$$(b) \quad X_w = (X_w)_F + (X_w)_{FUS} + (X_w)_T \quad (3-68)$$

where

$$(X_w)_F = \frac{1}{V_0} \left(\frac{\partial L_F}{\partial \alpha_F} \alpha - \frac{\partial D_F}{\partial \alpha_F} + L_F \right) \quad (3-69)$$

$$(X_w)_{FUS} = \frac{1}{V_0} \left(\frac{\partial \alpha_{FUS}}{\partial \alpha} \right) \left[\left(\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} + D_{FUS} \right) (\alpha - \varepsilon_{FUS}) - \frac{\partial D_{FUS}}{\partial \alpha_{FUS}} L_{FUS} \right] \quad (3-70)$$

$$(X_w)_T = \frac{1}{V_0} \left(\frac{\partial \alpha_T}{\partial \alpha} \right) \left[\frac{\partial L_T}{\partial \alpha_T} (\alpha - \varepsilon_T) - \frac{\partial D_T}{\partial \alpha_T} + L_T \right] \quad (3-71)$$

The Z-Force Derivatives

$$(a) \quad Z_u = (Z_u)_F + (Z_u)_{FUS} + (Z_u)_T \quad (3-72)$$

where

$$(Z_u)_F = - \left(\frac{\partial L_F}{\partial u_F} \alpha + \frac{\partial D_F}{\partial u_F} \alpha \right) \quad (3-73)$$

$$(Z_u)_{FUS} = - \left[\frac{\partial L_{FUS}}{\partial u_{FUS}} + \frac{\partial D_{FUS}}{\partial u_{FUS}} (\alpha - \varepsilon_{FUS}) \right] - \frac{\partial \alpha_{FUS}}{\partial u} \left[\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} + \frac{\partial D_{FUS}}{\partial \alpha_{FUS}} (\alpha - \varepsilon_{FUS}) + D_{FUS} \right] \quad (3-74)$$

$$(Z_u)_T = - \left[\frac{\partial L_T}{\partial u_T} + \frac{\partial L_T}{\partial \alpha_T} \frac{\partial \alpha_T}{\partial u} \right] \quad (3-75)$$

$$(b) \quad Z_w = (Z_w)_F + (Z_w)_{FUS} + (Z_w)_T \quad (3-76)$$

where

$$(Z_w)_F = - \frac{1}{V_0} \left(\frac{\partial L_F}{\partial \alpha_F} \right) \quad (3-77)$$

$$(Z_w)_{FUS} = - \frac{1}{V_0} \left(\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} \right) \left(\frac{\partial \alpha_{FUS}}{\partial \alpha} \right) \quad (3-78)$$

$$(Z_w)_T = - \frac{1}{V_0} \left(\frac{\partial L_T}{\partial \alpha_T} \right) \left(\frac{\partial \alpha_T}{\partial \alpha} \right) \quad (3-79)$$

The Pitching Moment (M) Derivatives

$$(a) \quad M_u = (X_u)_F^1 Z_F - (Z_u)_F^1 X_F - (Z_u)_T^1 X_T \\ + (X_u)_{TR}^1 Z_{TR} - (Z_u)_{TR}^1 X_{TR} + \frac{\partial M_{FUS}}{\partial u} + \frac{\partial M_{HUB_F}}{\partial u} \quad (3-80)$$

where

$$(X_u)_{TR} = - \left[\frac{\partial D_{TR}}{\partial u_{TR}} + \frac{\partial \alpha_{TR}}{\partial u} \left(\frac{\partial D_{TR}}{\partial \alpha_{TR}} \right) \right] \quad (3-81)$$

$$(Z_u)_{TR} = - \left[\frac{\partial D_{TR}}{\partial u_{TR}} - \frac{\partial \alpha_{TR}}{\partial u} \left(\frac{\partial D_{TR}}{\partial \alpha_{TR}} \right) \right] (\alpha - \epsilon_{TR}) \quad (3-82)$$

$$(b) \quad M_w = (X_w)_F^1 Z_F - (Z_w)_F^1 X_F - (Z_w)_T^1 X_T \\ - (Z_w)_{TR}^1 X_{TR} + \frac{1}{V_0} \left(\frac{\partial M_{FUS}}{\partial \alpha} + \frac{\partial M_{HUB_F}}{\partial \alpha} \right) \quad (3-83)$$

where
$$(Z_w)_{TR} = - \frac{1}{V_0} D_{TR} \quad (3-84)$$

(c)
$$M_q = Z_{w_T} \chi_T^2 - \frac{1}{Z_F} \left(\frac{\partial a_{1F}}{\partial q} L_F + \frac{\partial M_{HUB_F}}{\partial q} \right) \quad (3-85)$$

where
$$Z_{w_T} = - \frac{1}{V_0} \left(\frac{\partial L_T}{\partial \alpha_T} \right) \quad (3-86)$$

IV. SOLUTION OF THE CHARACTERISTIC EQUATION

In Chapter II it was shown that the longitudinal dynamics of the helicopter's motion could be described by three equations. These equations deal with forces in the X and Z directions and a moment, M, about the pitch axis. Knowing the stability derivatives, as outlined in Chapter III, the stage is set for determining the motions that characterize the helicopter's response to these forces and moments.

The procedure for obtaining the aircraft's modes of motion lies in a simultaneous solution to the three equations for longitudinal motion. A determinant is ideally suited for this purpose. What is sought is the aircraft's response to gravitational and aerodynamic forces and moments. Therefore, the stability derivatives are evaluated and the three equations set equal to zero and solved.

The determinant in question is derived from the two force equations and one moment equation that have been used for longitudinal dynamics. For the case of studying the helicopter's natural response (i.e., no forcing functions) the determinant is set equal to zero and the modes of motion are then found. If the helicopter's response to a given input is desired, then the determinant is set equal to that input (the forcing function) and solved. An example of such a forcing function is the step input of cyclic or collective movement.

The determinant derivation, from Reference 6, starts with the linearized equations of motion:

$$\frac{w}{g} \dot{u} = -w\theta \cos \tau + \Delta X \quad (4-1)$$

$$\frac{w}{g} \dot{w} - \frac{w}{g} v\dot{\theta} = -w\theta \sin \tau + \Delta Z \quad (4-2)$$

$$\beta \ddot{\theta} = \Delta M \quad (4-3)$$

ΔX and ΔZ are the forces arising from aerodynamics in disturbed flight and M is the aerodynamic moment in disturbed flight. $\dot{\theta} = q$.

It was shown in Chapter II that the changes in forces and moments arising from small disturbances could be written as Taylor series expansions. Thus a substitution for ΔX can be made as,

$$\Delta X = \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial q} q + \frac{\partial X}{\partial B_1} B_1 + \frac{\partial X}{\partial \theta_0} \theta_0 \quad (4-4)$$

where B_1 and θ_0 are cyclic and collective pitch control terms, respectively.

Using a shorthand notation of $\frac{\partial X}{\partial i} = X_i$, equation (4-4) can be written as:

$$\Delta X = X_u u + X_w w + X_q q + X_{B_1} B_1 + X_{\theta_0} \theta_0 \quad (4-5)$$

The linearized equations (4-1), (4-2), and (4-3) can now be written as:

$$\frac{w}{g} \dot{u} - X_u u - X_w w - X_q q + w\theta \cos \tau = X_{B_1} B_1 + X_{\theta_0} \theta_0 \quad (4-6)$$

$$-Z_u u + \frac{w}{g} \dot{w} - Z_w w - Z_q q - \frac{w}{g} v\dot{\theta} + w\theta \sin \tau = Z_{B_1} B_1 + Z_{\theta_0} \theta_0 \quad (4-7)$$

$$-M_u u - M_w w - M_{\dot{w}} \dot{w} + B\ddot{\theta} - M_q q = M_{B_1} B_1 + M_{\theta_0} \theta_0 \quad (4-8)$$

The resulting equations are linear ones with constant coefficients, and can be written in determinant form with Laplacian notation for compactness. With this notation S represents $d(\)/dt$. The resulting determinant is set equal to zero when the natural response of the aircraft is to be examined. (B_1 and θ_0 , the cyclic and collective pitch inputs = zero.) It should be noted that Z_q is always equal to zero and that M_w and X_q are negligibly small so that the final form of the stability determinant is:

$$\begin{vmatrix} S - X_u & -X_w & w_c \cos \tau \\ -Z_u & S - Z_w & -V_S \\ -M_u & -M_w & S^2 - M_q S \end{vmatrix} = 0 \quad (4-9)$$

Note: τ is usually a small angle such that $\cos \tau = 1$. For compatibility of units, velocity must be in feet per second. To convert knots to FPS, multiply by 1.6889.

A. THE CHARACTERISTIC EQUATION

The results of the determinant generated in the above manner is called the characteristic equation and in general is a quartic equation. It will be shown that in a hover the characteristic equation is reduced to a cubic equation.

The characteristic equation has the following form:

$$AS^4 + BS^3 + CS^2 + DS + E = 0 \quad (4-10)$$

where the value of the coefficients, A thru E, are determined by the determinant cross-product.

Solving the characteristic equation will yield four roots. In classical fixed-wing aircraft longitudinal analysis, the solution actually yields two sets of complex roots. These sets of roots describe two sinusoidal motions of the aircraft which are distinguished from each other by their periods. The longer period motion is called the phugoid and the shorter period motion is called the short period.

The fixed-wing characteristic equation quartic can also be represented by the product of two quadratic equations. The coefficients of the two quadratic equations contain terms which define the damping ratio, ζ , and the response frequency, ω , of the modes of motion associated with the quadratic equations.

$$AS^4 + BS^3 + CS^2 + DS + E = \left\{ S^2 + 2\zeta_P\omega_P S + \omega_P^2 \right\} \left\{ S^2 + 2\zeta_{SP}\omega_{SP} S + \omega_{SP}^2 \right\} \quad (4-11)$$

where the subscript P defines that motion associated with the long period motion of the aircraft, or phugoid, and the subscript SP is used to denote the short period motion.

The phugoid mode of motion is one in which the aircraft's angle of attack remains essentially constant while airspeed and altitude change as aircraft kinetic energy (airspeed) and potential energy (altitude) are exchanged until the aircraft's motion dampens out at trim airspeed (when the system is convergent) or until the aircraft departs controlled flight (if the phugoid has a divergent nature) (see Figure 4-1). The respective velocities and altitudes at points 1, 2, and 3 would be increasing or decreasing depending on whether the aircraft's oscillation was damped or divergent as shown in Figure 1-2.

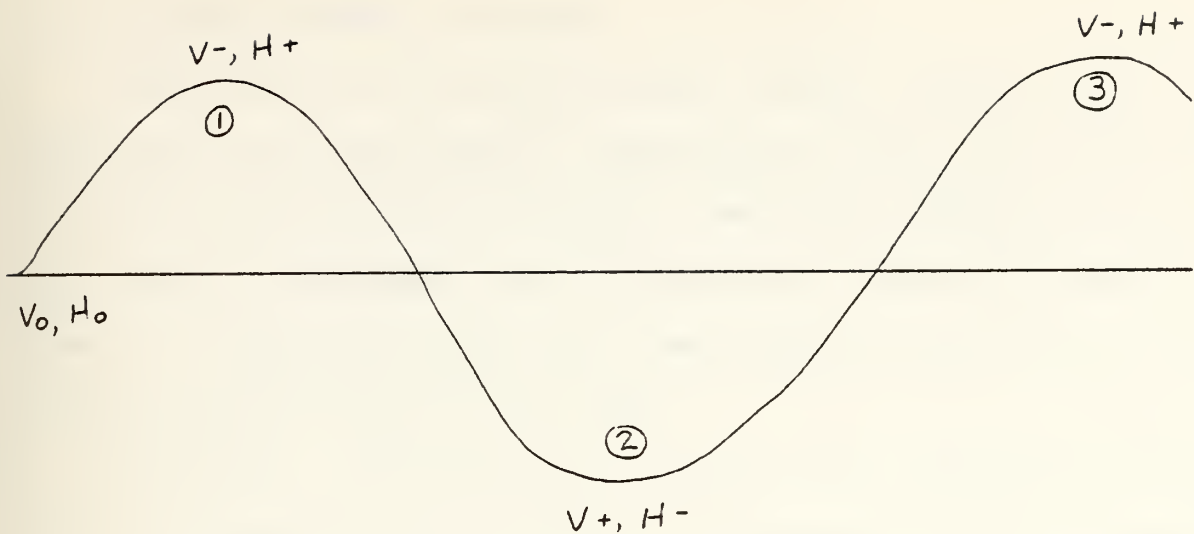


Figure 4-1. Phugoid Response

For helicopters the period of the phugoid is typically long, on the order of 30 seconds while the short period mode of motion is a heavily damped one in which the aircraft's velocity remains constant. Since the short period is heavily damped, its period is typically on the order of one second.

The aircraft's resultant motion can be compared to a spring-mass-damper system which has two springs and two dampers. Keeping this in mind, ζ , the damping ratio, is a measure of the amount of damping in the system. Insofar as the short period has a more heavily damped motion, it follows that the value of ζ_{sp} will be higher than that for ζ_p .

B. HELICOPTER RESPONSE IN A HOVER

Unfortunately, the helicopter's modal response cannot be given the same clear, consistent physical interpretation as that for a fixed-wing aircraft. One reason for this becomes immediately apparent when hovering flight is considered. For a conventional fixed-wing aircraft all phases of flight are conducted with some forward flight velocity. This is obviously not so with the helicopter.

Surprisingly, the characteristic equation for a helicopter in hover is a cubic equation, not a quartic. The reason for this is found by examining the Z-force equation in the determinant:

$$\begin{vmatrix} Z_u & S-Z_w & V_0 S \end{vmatrix} = 0 \quad (4-12)$$

V_0 equals zero in a hover, and if Z_u equals zero for hovering flight (as it frequently does) the vertical motion is immediately decoupled from the pitching motions and fore and aft motions. As a result, the vertical motion is entirely dependent on Z_w and is usually a heavily damped subsidence motion since Z_w is usually a large negative number. The assertion that the vertical motion of the helicopter in a hover is very dependent on Z_w is borne out by the fact that a helicopter's vertical motion is known to be very responsive to vertical gusts of wind and to collective pitch inputs.

Once the solution to the Z-force equation is known ($S = Z_w$), it can be removed from the determinant and the system of equations can then be reduced to a 2 X 2 determinant.

$$\begin{vmatrix} S - X_u & W \\ -M_u & S^2 - M_q S \end{vmatrix} = 0 \quad (4-13)$$

and the resulting characteristic equation will be a cubic:

$$S^3 - (X_u + M_q) S^2 + X_u M_q S + M_u W = 0 \quad (4-14)$$

The solution of the cubic equation will generally yield a negative real root and a positive complex root pair. The negative real root indicates a stable convergence and is principally due to pitch damping of the rotor whereas the positive pair of complex roots indicates unstable oscillation and is due to the coupling of the pitch and longitudinal velocity by the speed stability derivative M_u .

For an articulated rotor, the real root (which indicates the short period mode) typically has a time to half amplitude of $T_{\frac{1}{2}} = 1$ to 2 seconds. The long period mode is represented by the oscillatory root and has a period of $T = 10$ to 20 seconds and since it is a divergent motion, a time to double amplitude of $T_2 = 3$ to 4 seconds (Ref. 10).

Although the phugoid motion is unstable, the period and time to double amplitude are sufficiently long for the pilot to observe the helicopter's reactions and make necessary control movements to maintain control of the aircraft.

Hingeless rotors have a higher degree of pitch damping than articulated rotors. This high degree of damping serves to greatly increase

the magnitude of the real root and it also increases the period and time to double amplitude of the oscillatory mode. For hingeless rotors the time to half amplitude is typically 0.2 to 0.5 seconds, while the oscillatory phugoid mode has a period of 10 to 20 seconds with a time to double amplitude of 10 to 15 seconds.

C. APPROXIMATION OF THE SHORT PERIOD DURING HOVER

The initial response of the helicopter to gusts is primarily that of vertical and pitch acceleration, with little longitudinal acceleration. Since this analysis assumes perturbations from steady, level flight, the longitudinal degree of freedom can be neglected in order to approximate the short period mode. (It should be noted that the short period mode is heavily damped and is characterized by near zero velocity change, so that the neglect of longitudinal acceleration is reasonable for this analysis.)

With this assumption, the stability determinant is reduced to:

$$\begin{vmatrix} -M_w & S-M_q \\ S-Z_w & -V \end{vmatrix} = 0 \quad (4-15)$$

and the characteristic equation becomes:

$$(S - M_q) (S - Z_w) - M_w V = 0 \quad (4-16)$$

It has been explained that in hover the pitch and vertical motions decouple and the two solutions of the characteristic equations are:

$$S = Z_w \quad \text{and} \quad S = M_q \quad (4-17)$$

As a check on the validity of this assertion note the following comparison in Table 4-1 between stability derivatives and roots from the solution of the stability quartic for three different helicopters. The comparison shows generally good correlation.

TABLE 4-1. Comparison

	Z_w	M_q	ROOTS
B0-105	-0.3317	-3.3972	-0.331, -3.4521
CH-53D	-0.298	-0.499	-0.2934, -0.8232
OH-6	-0.3404	-1.7645	-0.3544, -1.8794

The helicopter has neutral static stability when disturbed by perturbations in pitch or roll. This is because no moments are generated directly by these motions to move the helicopter back to or away from its equilibrium position. However, the helicopter in hover does possess positive static stability when disturbed by longitudinal or lateral perturbations of wind velocity.

D. HELICOPTER RESPONSE IN FORWARD FLIGHT

As the helicopter departs hovering flight and transitions to forward flight, new forces come into existence which start to change the stability picture. These forces arise from the increasing dynamic pressure being built up on the aerodynamic surfaces of the helicopter. As a

result the dynamics of the helicopter in forward flight are different from those characteristics exhibited in hover. The forces and moments that act on the helicopter are contributed by the main rotor, tail rotor, fuselage and tail surface aerodynamics, and gravity.

A stability derivative that arises from forward flight is M_w , the pitching moment due to angle of attack perturbation. This derivative is usually equal to zero for the hover condition. It has an unstabilizing effect on the helicopter's motion for the following reason: As the helicopter accelerates, the advancing blade experiences increased dynamic pressure. This increase in pressure results in a change in the blade angle of attack which produces a lateral moment on the rotor disc (toward the retreating blade). The moment thus generated is proportional to forward velocity. Because of the gyroscopic effect of the rotating disc, the tip path plane will respond to this moment after 90 degrees of movement. Therefore the rotor disc will be tilted aft. The angle of attack increase results in a pitch up moment of the aircraft which further increases the angle of attack on the rotor system. Therefore the dynamics of the rotor are a source of instability for the helicopter in forward flight.

To counter the angle of attack instability of the main rotor in forward flight, a horizontal tail can be incorporated on the helicopter. The forces and moments produced by the horizontal tail are proportional to forward velocity so that they are approximately zero in hover and increase with speed. The horizontal tail will have a stabilizing influence on the helicopter's motions in much the same way as a horizontal tail on a fixed-wing aircraft. Since the moments produced by both the

main rotor and horizontal tail are proportional to velocity, their relative contributions to stability are actually independent of speed.

As can be seen, M_w is influenced by two opposing sources, an unstable contribution from the rotor and a stabilizing contribution from the horizontal tail. Without a horizontal tail, the dynamics of the helicopter in forward flight are characterized by two stable damped motions (from the negative real roots) and an unstable oscillatory mode (from a positive pair of complex roots). In this configuration the flying qualities are degraded due to the angle of attack instability.

The dynamics of the helicopter in forward flight can be changed by the addition of a large enough horizontal tail such that static stability is achieved. For the case of a large horizontal tail, the pitch and vertical real roots are transformed into two oscillatory roots with a short period and high damping. The latter motion is similar to that found in a fixed-wing aircraft.

In actual practice other considerations must be taken into account which may eclipse the goal of achieving fixed-wing-like dynamics in forward flight. One such limitation is that a horizontal tail which is large enough to counter the unstabilizing influences of the main rotor may simply be too large for weight or drag considerations. Another factor to consider is that tail effectiveness is reduced at low speeds due to interference with rotor and fuselage wakes. In spite of these problems it should be noted that almost all single rotor helicopters do have horizontal tail planes because of the improvement in flying qualities that the addition of this component provides.

E. APPROXIMATION OF THE SHORT PERIOD IN FORWARD FLIGHT

The assumptions made for the analysis of the short period motion in a hover are also valid for the approximation of the short period motion in forward flight. Of course the characteristic equation (4-15) is not solved so readily because of the $-M_w V$ term. In a hover this term went to zero, but in forward flight the characteristic equation must be solved.

$$(S - M_q) (S - Z_w) = M_w V \quad (4-18)$$

Typically, the short period motion of the helicopter is characterized by two negative real roots. This, of course, means that the short period is a stable motion and is heavily damped. Because of the action of the horizontal tail as explained above, certain helicopters do occasionally exhibit a stable oscillatory mode, more in keeping with fixed-wing dynamics.

V. INTERPRETATION OF THE STABILITY DERIVATIVES

It has been shown how the individual stability derivatives can be calculated knowing some basic data about the helicopter, and having access to theoretical charts relating aircraft performance and response parameters. An explanation of the characteristics of these derivatives and their effects on the helicopter's motion is now in order. To begin with it should be noted that since all airframe contributions are proportional to airspeed, the fuselage and horizontal tail forces will be equal to zero at zero flight velocity. This fact serves to simplify certain calculations for the hover condition.

A. X_u = DRAG DAMPING

This stability derivative acts as a damping force. It will be negative in sign. The interpretation of X_u is that it represents an increase in drag with an increase in forward flight velocity. Physically this is seen to be true because as forward speed increases the thrust vector (and the rotor disk) must be tilted more forward to overcome the effects of increased drag.

In relation to the dynamic motions, X_u has a weak effect on the phugoid, but one which tends to make the phugoid more stable.

B. X_w = DRAG DUE TO ANGLE OF ATTACK

X_w can be interpreted as the change in drag on the aircraft which is brought about by change in angle of attack. It is usually of small value and does not have much influence on either the static or dynamic characteristics of stability.

X_w is usually very small or zero for hovering flight. For the purposes of simplifying calculations, it can safely be assumed to equal zero for the hover condition.

C. Z_u = LIFT DUE TO VELOCITY

This stability derivative is always negative for fixed-wing aircraft and corresponds to increased lift at higher velocities. (Remember, the Z-axis is positive downward.) This is not so for helicopters, however. According to Reference 9, for helicopters Z_u is negative at low speeds but positive at high speeds. This is not especially significant except at higher forward velocity where it might affect the dynamic divergence. Z_u is usually small for an articulated rotor.

Like X_w , Z_u can be assumed to be equal to zero for the hover condition and for the same reasons.

D. Z_w = VERTICAL DAMPING

Z_w acts as a damping force in the same manner that X_u does. It also is negative in sign and occurs because of the vertical motions of the aircraft. For a helicopter this is an important parameter, especially in a hover where it describes the response of the aircraft to vertical gusts. This stability derivative is nearly independent of airspeed for the helicopter whereas it is proportional to airspeed for fixed-wing aircraft.

E. M_u = VELOCITY STABILITY

This parameter describes the pitch tendencies of the aircraft with respect to speed changes. According to Reference 9, at hover and for very low speeds, most helicopter configurations have M_u positive. This leads to a positive stick gradient. A positive value for M_u could lead to oscillatory instability and it indicates the aircraft is sensitive to turbulence.

M_u changes sign at high forward flight velocities. Negative values of M_u lead to dynamic divergence.

F. M_w = ANGLE OF ATTACK STABILITY

M_w indicates the tendency of the aircraft to pitch up or down as angle of attack is increased. A negative value of M_w is stabilizing as it tends to return the aircraft to its previous position whereas a positive value would be divergent in nature. Most helicopters exhibit values of M_w that are neutral or positive. A positive value will lead to dynamic divergence in forward flight. Seckel claims that the center of gravity position can have an effect on the values of M_w and consequently on the resultant stability characteristics of the aircraft. If the fuselage and horizontal tail contributions are stable with respect to angle of attack, moving the center of gravity forward will make the aircraft more stable and aft less so. Conversely, if the fuselage and horizontal tail have destabilizing tendencies, then the opposite will occur.

According to Reference 10, M_w receives an unstable contribution from the rotor and fuselage and a stable contribution from the horizontal

tail. M_w is the third stability derivative which can be assumed to be equal to zero for hover calculations with no loss of accuracy in the results.

G. M_q = PITCH DAMPING

This derivative is negative and is a very important one for response to control deflection and dynamic stability. Most helicopters require some augmentation of this angular damping for good handling qualities. The numerical value of M_q should be less than -0.5 [Ref. 9]. Values of -2.0 are better, but the degree of improvement decreases for values less than -2.0.

VI. ANALYSIS OF RESULTS

As can be seen from the sample problem, Appendix A, the stability characteristics of the helicopter under investigation consisted of a pair of damped oscillatory roots, a divergent real root, and a convergent real root. These characteristics differ from the low-speed response for the same helicopter. At advance ratios of 0.1 to 0.2 this aircraft's response was identical to that expected for helicopters in low speed flight, namely two heavily damped stable roots and a pair of divergent oscillatory roots. Thus, the stability characteristics of the helicopter have changed at high speed.

The resulting motion of the helicopter at high advance ratios cannot be accurately predicted, however, as can be seen from Table 6-1. This table shows the different response modes of five helicopters at various conditions of airspeed and center of gravity location. Some interesting observations can be made from this table.

The response of some helicopters may not change at all with airspeed. The CH-53D is an example of this. Data obtained from Reference 11 indicates the CH-53 retains the response mode characteristics associated with hovering flight out to at least 140 knots. Different responses are exhibited by other helicopters. The OH-6 has a typical hover response mode which changes into two damped real roots and a damped oscillatory motion prior to 100 knots. The BO-105 also changes from the hover response at forward speed but not until an airspeed greater than 100

TABLE 6-1. RESPONSE MODE COMPARISON

AIRCRAFT	ROOTS		
	HOVER	100 KNOTS	140 KNOTS
CH-53D	0.1139 ± 0.4705	0.1648 ± 0.3097	0.2538 ± 0.2769
	-0.2934	-0.3374	-0.2793
	-0.8232	-1.3822	-1.7994
	HOVER	100 KNOTS	130 KNOTS
OH-6	0.0516 ± 0.4658	-0.0192 ± 0.3186	-0.1027 ± 0.3729
	-0.3544	-1.0092	-1.0033
	-1.8794	-2.6212	-2.8293
	HOVER	100 KNOTS	145 KNOTS
BO-105	0.0188 ± 0.4333	0.2709 ± 0.4673	1.2449
	-0.331	-0.4040	0.3999
	-3.4521	-4.7375	-0.3189
			-5.9681
	HOVER	100 KNOTS	130 KNOTS
UH-1H		(MID C.G.)	
	0.1685 ± 0.3535	-0.0060 ± 0.2820	-0.0211 ± 0.2392
	-0.4577 ± 0.1527	-0.8653 ± 1.0150	-0.9354 ± 1.4882
		(AFT C.G.)	
		0.0220 ± 0.2880	
		-0.8720 ± 0.7692	
	HOVER	100 KNOTS	140 KNOTS
AH-1G		(AFT C.G.)	
	0.1205 ± 0.2617	0.0089 ± 0.2666	-0.0337 ± 0.2051
	-0.4323 ± 0.1989	-0.7179 ± 0.6312	-0.7442 ± 1.5398

knots and then the response is represented by four real roots, two convergent and two divergent.

Still other examples of stability can be seen with the UH-1H and the AH-1G. Both of these helicopters have stable oscillatory periods typical of the response of a fixed-wing airplane. Of interest here is the change in response with change in center of gravity position at 100 knots. Moving the center of gravity can have a profound effect of a helicopter. For both the UH-1H and the AH-1G, moving the center of gravity aft has a destabilizing influence.

APPENDIX A
SAMPLE PROBLEM

MAIN ROTOR

$$R = 24 \text{ ft}$$

$$\Omega = 29 \text{ rad/sec}$$

$$b = 4$$

$$e = 0.5 \text{ ft}$$

$$\theta_1 = -8^\circ$$

$$c = 1.75 \text{ ft}$$

$$a = 5.73/\text{rad}$$

$$M_S = 85.4 \text{ slug-ft}$$

$$I_b = 1200 \text{ slug-ft}^2$$

$$l_{Z_F} = -6 \text{ ft}$$

$$l_{X_F} = -10 \text{ ft}$$

FUSELAGE

$$W = 10,000 \text{ lbs}$$

$$I_y = 17,500 \text{ slug-ft}^2$$

$$l_{FUS} = 50 \text{ ft}$$

$$A_X = 60 \text{ ft}^2$$

$$A_Z = 200 \text{ ft}^2$$

TAIL ROTOR

$$R = 4.6 \text{ ft}$$

$$b = 4$$

$$c = 0.75 \text{ ft}$$

$$\theta_1 = 0^\circ$$

$$\Omega = 146.6 \text{ rad/sec}$$

$$a_{TR} = 5.73/\text{rad}$$

$$l_{X_{TR}} = -30 \text{ ft}$$

$$l_{Z_{TR}} = -6 \text{ ft}$$

$$i = 0$$

HORIZONTAL TAIL

$$S = 20 \text{ ft}^2$$

$$a_T = 3.5/\text{rad}$$

$$\text{Airfoil} = \text{NACA 0015}$$

$$AR = 4.0$$

$$l_{X_T} = -20 \text{ ft}$$

$$C_{D0} = 0.02$$

$$i_T = 0$$

$$l_{Z_T} = 0$$

The helicopter's stability characteristics will be evaluated at $V_0 = 203 \text{ FPS}$ and at sea level. Using the data supplied for the helicopter, and the definitions listed in Chapter III or the program STAB from Appendix B, the following values are determined:

$$\rho_{SL} = 0.002378 \text{ slugs/ft}^3$$

$$q_0 = 49.0 \text{ psi}$$

$$T.F. = 2,084,504 \text{ lbs}$$

$$\sigma = 0.093$$

$$m = 310.8 \text{ slugs}$$

$$\mu = 0.292$$

$$\Omega R = 696 \text{ fps}$$

$$M_{TIP} = 0.805$$

$$V_{TIP} = 899 \text{ fps}$$

$$(T.F.)_{TR} = 71,896$$

$$\sigma_{TR} = 0.207$$

$$(\Omega R)_{TR} = 674 \text{ fps}$$

$$\mu_{TR} = 0.30$$

$$(M_T)_{TR} = 0.783$$

Use Figure A-1 to obtain values for $C_{L_{FUS}}$ and $C_{D_{FUS}}$ for $\alpha_{FUS} = 0$.

$$C_{L_{FUS}} = -0.005$$

$$C_{D_{FUS}} = 0.16$$

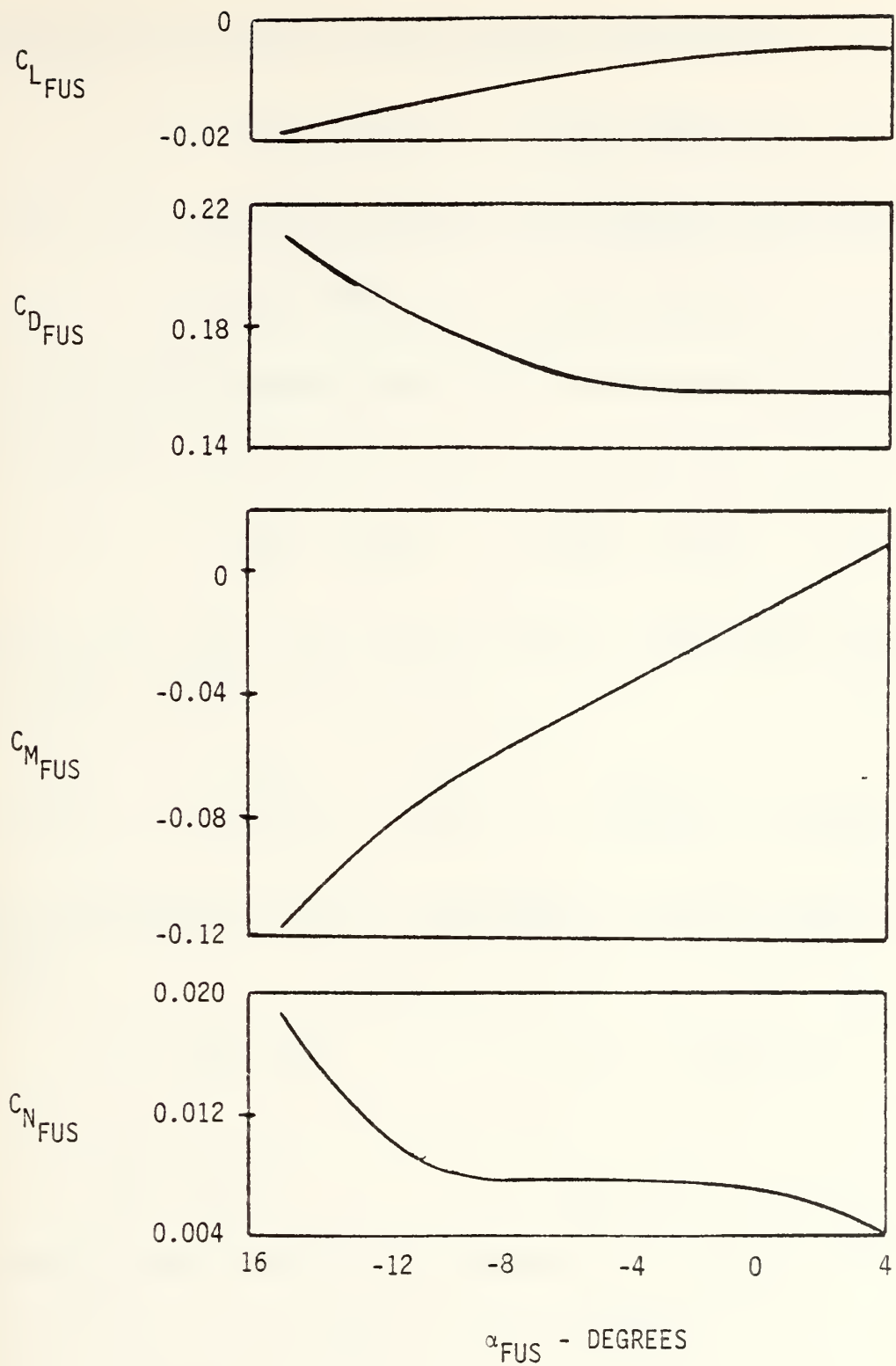


Figure A-1. Fuselage Characteristics for the Sample Single Rotor Helicopter ($\beta_s = 0$)

Equations (3-1) and (3-2) yield the fuselage lift and drag:

$$L_{FUS} = -49 \text{ lb} \quad D_{FUS} = 470.4 \text{ lb}$$

Main rotor lift and drag are obtained from equations (3-3) and (3-4).

$$L_F = 10049 \text{ lb} \quad D_F = -470.4 \text{ lb}$$

The main rotor lift and drag coefficients are calculated from equations (3-5) and (3-6).

$$\frac{C_L'}{\sigma} = 0.0473 \quad \frac{C_D'}{\sigma} = -0.0022$$

Equations (3-7), (3-8), and (3-9) are used to correct these values if the main rotor solidity differs from 0.1.

$$\Delta\sigma = -0.007 \quad \left(\frac{C_L'}{\sigma}\right)_{0.1} = 0.0473 \quad \left(\frac{C_D'}{\sigma}\right)_{0.1} = -0.0021$$

Now use the appropriate charts in Reference 6 to obtain values for the following:

$$a_1 = 0.0611 \text{ rad} \quad \alpha_c = -0.1222 \text{ rad}$$

$$\frac{C_Q}{\sigma} = 0.0025 \quad \theta_{.75} = 5^\circ$$

Also use the charts in Section 5.3 of Reference 5 to obtain:

$$\lambda = -0.045 \quad a_0 = 2.3^\circ$$

The angle of attack of the main rotor and the main rotor torque are calculated from equations (3-10) and (3-11).

$$\alpha_c = -0.1241 \text{ rad} \quad Q = 11632 \text{ ft-lb}$$

The downwash interference factors for this configuration are $K_{FFUS} = K_{FT} = K_{FTR} = 1.0$ and the downwash interference angles equal 0.0312 radians from equation (3-13).

A relation between α_{FUS} and $C_{M_{FUS}}$ is needed to plot against the experimentally obtained fuselage pitching moment data. Equation (3-14) will yield:

$$\alpha_{FUS} = -0.0164 + 0.5911 C_{M_{FUS}}$$

The point of intersection on Figure A-2 will yield the fuselage trim angle of attack, -2 degrees or -0.0349 radians. Using this value use Figure A-1 again to obtain the following new values for the parameters indicated:

$$C_{L_{FUS}} = -0.0065 \quad C_{M_{FUS}} = -0.025$$

$$C_{D_{FUS}} = 0.158 \quad C_{N_{FUS}} = 0.007$$

Use equations (3-1) and (3-2) to recalculate L_{FUS} and D_{FUS} . Also use equations (3-15) and (3-16) to determine M_{FUS} and N_{FUS} , respectively.

$$L_{FUS} = -63.7 \text{ lb} \quad M_{FUS} = -3675 \text{ ft-lb}$$

$$D_{FUS} = 464.5 \text{ lb} \quad N_{FUS} = 1029 \text{ ft-lb}$$

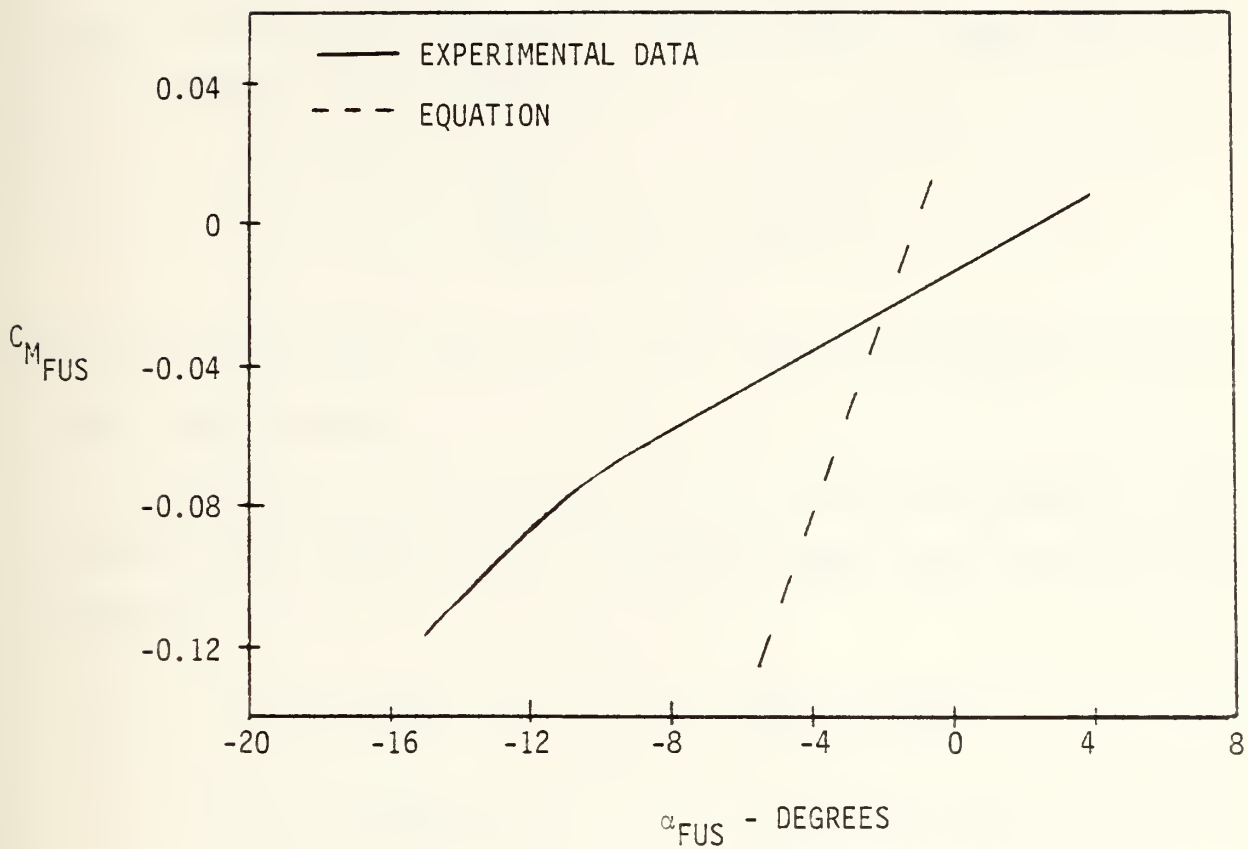


Figure A-2. Superposition of the Calculated and the Experimental Fuselage Pitching Moment Data

Using N_{FUS} and Q_F determine the tail rotor thrust and the tail rotor lift coefficient from equations (3-17) and (3-18).

$$T_{TR} = 422 \text{ lb} \qquad \left(\frac{C_L'}{\sigma} \right)_{TR} = 0.0293$$

Enter the charts in Reference 6 to obtain additional tail rotor parameters, knowing the tail rotor lift coefficient. Blade twist = 0, $\mu = 0.3$, and $M_T = 0.8$. Assume $\alpha_C = 0$.

$$\left[\frac{C_D'}{\sigma} \right]_{TR} \bigg|_{0.1} = 0.0015$$

Equations (3-19) and (3-20) will yield a value for the drag of the tail rotor. $D_{TR} = 28.288$.

From the trim values obtained earlier and using equations (3-21) through (3-26) determine the following values for the horizontal tailplane.

$$\begin{aligned} \alpha &= -0.0037 \text{ rad} & C_{D_T} &= 0.0216 \\ \alpha_T &= -0.0349 \text{ rad} & L_T &= -12.7 \text{ lb} \\ C_{L_T} &= -0.1396 & D_T &= 19.6 \text{ lb} \end{aligned}$$

Equations (3-27) through (3-30) will yield a better approximation of the main rotor lift and drag.

$$L_F = 10060 \text{ lbs} \qquad D_F = -515.2 \text{ lbs}$$

Use equations (3-5) through (3-9) to obtain better estimates for the main rotor lift and drag coefficients. The final trim values of the helicopter are:

$$\mu = 0.3 \qquad \left(\frac{C_L'}{\sigma}\right) = 0.053$$

$$M_T = 0.8 \qquad \left(\frac{C_D'}{\sigma}\right) = -0.0026$$

$$\sigma = 0.092 \qquad L_F = 10060 \text{ lbs}$$

$$\theta_1 = -8^\circ \qquad D_F = -515.2 \text{ lbs}$$

$$\theta_{.75} = -5^\circ \qquad \lambda_F = -0.045$$

The trim values for $\frac{C_L'}{\sigma}$, $\frac{C_D'}{\sigma}$, and $\theta_{.75}$ are used to enter the charts in Section 7.5 of Reference 5 to get the nondimensional isolated derivatives for the front rotor.

$$\left[\frac{\partial\left(\frac{C_L'}{\sigma}\right)}{\partial\mu}\right]_F = -0.02 \qquad \left[\frac{\partial\left(\frac{C_D'}{\sigma}\right)}{\partial\mu}\right]_F = 0.014$$

$$\left[\frac{\partial\lambda}{\partial\mu}\right]_F = -0.05 \qquad \left[\frac{\partial a_1}{\partial\mu}\right]_F = 0.17$$

$$\left[\frac{\partial\left(\frac{C_L'}{\sigma}\right)}{\partial\alpha_c}\right]_F = 0.38 \qquad \left[\frac{\partial\left(\frac{C_D'}{\sigma}\right)}{\partial\alpha_c}\right]_F = 0.07$$

$$\left[\frac{\partial \lambda}{\partial \alpha_c} \right]_F = 0.2$$

$$\left[\frac{\partial a_1}{\partial \alpha_c} \right]_F = 0.24$$

Solidity corrections are made to these values, if necessary, using equations (3-31) through (3-41).

Using the isolated derivatives just obtained, the main rotor local derivatives can be calculated. Computation is done by equations (3-42) through (3-49). The following results will be obtained:

$$\frac{\partial L_F}{\partial u_F} = -5.56 \frac{\text{lb-sec}}{\text{ft}}$$

$$\frac{\partial L_F}{\partial \alpha_F} = 73666 \frac{\text{lb}}{\text{rad}}$$

$$\frac{\partial D_F}{\partial u_F} = 3.89 \frac{\text{lb-sec}}{\text{ft}}$$

$$\frac{\partial D_F}{\partial \alpha_F} = 13570 \frac{\text{lb}}{\text{rad}}$$

$$\frac{\partial a_{1F}}{\partial u_F} = 0.000244 \frac{\text{rad-sec}}{\text{ft}}$$

$$\frac{\partial a_{1F}}{\partial \alpha_F} = 0.24$$

$$\frac{\partial M_{HUB}}{\partial u_F} = 17.54 \text{ lb-sec}$$

$$\frac{\partial M_{HUB}}{\partial \alpha_F} = 17237 \frac{\text{lb-ft}}{\text{rad}}$$

$$\frac{\partial a_{1F}}{\partial q} = -0.09881$$

Use Figure A-1 to obtain the partial derivative values of C_L , C_D , and C_M with respect to α by obtaining the slope of the line at the trim value of α_{FUS} (-0.0312). The following values were obtained:

$$\frac{\partial C_{L_{FUS}}}{\partial \alpha_{FUS}} = \frac{0.0005}{\text{degree}} = \frac{0.0287}{\text{radian}}$$

$$\frac{\partial C_{D_{FUS}}}{\partial \alpha_{FUS}} = \frac{-0.0005}{\text{degree}} = \frac{-0.0287}{\text{radian}}$$

$$\frac{\partial C_{M_{FUS}}}{\partial \alpha_{FUS}} = \frac{0.0058}{\text{degree}} = \frac{0.3323}{\text{radian}}$$

The fuselage local derivatives can now be calculated using equations (3-50) through (3-55). The following results will be obtained:

$$\frac{\partial L_{FUS}}{\partial u_{FUS}} = -0.628 \frac{\text{lb-sec}}{\text{ft}}$$

$$\frac{\partial L_{FUS}}{\partial \alpha_{FUS}} = 280.8 \frac{\text{lb}}{\text{rad}}$$

$$\frac{\partial D_{FUS}}{\partial u_{FUS}} = 4.576 \frac{\text{lb-sec}}{\text{ft}}$$

$$\frac{\partial D_{FUS}}{\partial \alpha_{FUS}} = -84.23 \frac{\text{lb}}{\text{rad}}$$

$$\frac{\partial M_{FUS}}{\partial u_{FUS}} = -36.21 \text{ lb-sec}$$

$$\frac{\partial M_{FUS}}{\partial \alpha_{FUS}} = 48848 \frac{\text{lb-ft}}{\text{rad}}$$

Values for the horizontal tailplane derivatives are calculated using the previously obtained values for L_T and D_T and equations (3-56) through (3-59).

$$\frac{\partial L_T}{\partial u_T} = -0.1251 \frac{\text{lb-sec}}{\text{ft}}$$

$$\frac{\partial L_T}{\partial \alpha_T} = 3430 \frac{\text{lb}}{\text{rad}}$$

$$\frac{\partial D_T}{\partial u_T} = 0.1931 \frac{\text{lb-sec}}{\text{ft}}$$

$$\frac{\partial D_T}{\partial \alpha_T} = -7.07 \frac{\text{lb}}{\text{rad}}$$

The tail rotor derivative is obtained from equation (3-20a) and is

$$\frac{\partial D_{TR}}{\partial u_{TR}} = 0.1545 \frac{\text{lb-sec}}{\text{ft}}$$

$K_{FT} = 1.0$ for this configuration and $K_{RF} = 0$, therefore:

$$\frac{\partial \alpha_{FUS}}{\partial \alpha} = \frac{\partial \alpha_T}{\partial \alpha} = 0.685 \qquad \frac{\partial \alpha}{\partial u} = 0$$

$$\frac{\partial \alpha_{FUS}}{\partial u} = \frac{\partial \alpha_T}{\partial u} = 0.000513 \qquad \frac{\partial \alpha_F}{\partial \alpha} = 1.0$$

Equations (3-64) through (3-85) are used to calculate the aircraft total stability derivatives. Alternatively, the computer programs in Appendix B will simplify this process. XUXW can be used to determine the X-force derivatives, ZUZW to find the Z-force derivatives, and MUWQ to find the pitching moment derivatives. Values obtained by either method are as follows:

$$X_u = -8.6318 \frac{\text{lb-sec}}{\text{ft}}$$

$$M_u = -4.783 \frac{\text{sec}}{\text{ft}}$$

$$X_w = -19.075 \frac{\text{lb-sec}}{\text{ft}}$$

$$M_w = 307.32 \frac{\text{sec}}{\text{ft}}$$

$$Z_u = 4.344 \frac{\text{lb-sec}}{\text{ft}}$$

$$M_q = -17838.2 \frac{\text{sec-ft-lb}}{\text{rad}}$$

$$Z_w = -375.41 \frac{\text{lb-sec}}{\text{ft}}$$

MUWQ incorporates some additional relations which must be known if equations (3-80) through (3-85) are being hand calculated.

$$\frac{\partial M_{FUS}}{\partial u} = \frac{\partial M_{FUS}}{\partial u_{FUS}} + \frac{\partial M_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial u}$$

$$\frac{\partial M_{HUB}}{\partial u} F = \frac{\partial M_{HUB}}{\partial u_F} F + \frac{\partial M_{HUB}}{\partial \alpha_F} F \frac{\partial \alpha_F}{\partial u}$$

$$\frac{\partial M_{FUS}}{\partial \alpha} = \frac{\partial M_{FUS}}{\partial \alpha_{FUS}} \frac{\partial \alpha_{FUS}}{\partial \alpha}$$

$$\frac{\partial M_{HUB}}{\partial \alpha} F = \frac{\partial M_{HUB}}{\partial \alpha_F} F \frac{\partial \alpha_F}{\partial \alpha}$$

$$\frac{\partial M_{HUB_F}}{\partial q} = \frac{\partial M_{HUB_F}}{\partial u} l_{Z_F} - \frac{1}{V_0} \frac{\partial M_{HUB_F}}{\partial \alpha} l_{X_F} + \frac{eb\Omega^2 M_S}{2} \left(\frac{da_F^1}{dq} \right)$$

Divide the force derivatives by mass and the moment derivatives by pitching moment of inertia (I_y) to form the stability determinant equation (4-9).

$$\begin{vmatrix} S + .0278 & 0.0614 & 32.2 \\ -0.014 & S + 1.2079 & -203 S \\ 0.0003 & -0.0176 & S^2 + 1.019 S \end{vmatrix} = 0$$

The characteristic equation will be:

$$S^4 + 2.255 S^3 - 2.2788 S^2 - 0.0776 S - 0.0037 = 0$$

Solving the quartic equation will yield the following roots:

$$0.7843$$

$$-3.0049$$

$$-0.0172 \pm 0.0357$$

Therefore this helicopter has the characteristics of a heavily damped root, a divergent motion, and a damped oscillatory motion.

The time to half amplitude of the damped root is:

$$\tau_{1/2} = \frac{.69}{|-3.0049|} = 0.230 \text{ sec.}$$

The time to double amplitude of the divergent root is:

$$\tau_2 = \frac{.69}{|0.7843|} = 0.880 \text{ sec.}$$

The time to half amplitude for the oscillatory mode is:

$$\tau_{1/2} = \frac{.69}{|-0.0172|} = 40.12 \text{ sec.}$$

WORKSHEET FOR STAB

STORAGE	PARAMETER	VALUE
01	b	4
02	c	1.74
03	R	24
04	Ω	29
05	V	203
06	k	10000
07	ρ	.002378
08	I_b	1200
09	a	5.73
10	V_S	116.4
11	M_S	85.4
12	e	.5

STORAGE	PARAMETER	VALUE
	ΩR	696
	$\sigma = \frac{bc}{\pi R}$	.093
	$\mu = \frac{v}{\Omega R}$	.2917
	$m = \frac{W}{32.2}$	310.8
	T.F	2084504
	γ	6.5928
	$V_T = \Omega R + V$	899
	$M_T = \frac{V_T}{V_S}$	.805
	$\frac{eb\Omega^2 M_S}{2}$	71821
	$q_0 = \frac{1}{2}\rho V^2$	49
	T.F. $\cdot\sigma$	193,526
	$\frac{T.F.\cdot\sigma}{\Omega R}$	278

WORKSHEET FOR XUXW

STORAGE	DERIVATIVE	VALUE
01	L_F	10060
02	L_{FUS}	-63.7
03	D_{FUS}	464.5
04	L_T	-12.7
05	V_0	203
09	α	-.0037
10	$\alpha^{-\varepsilon}$	-.0349
12	$\frac{\partial L_F}{\partial u_F}$	-5.56
14	$\frac{\partial L_F}{\partial \alpha_F}$	73666
15	$\frac{\partial D_F}{\partial \alpha_F}$	13570
16	$\frac{\partial L_{FUS}}{\partial u_{FUS}}$	-.628
17	$\frac{\partial D_{FUS}}{\partial u_{FUS}}$	4.576

STORAGE	DERIVATIVE	VALUE
18	$\frac{\partial L_{FUS}}{\partial \alpha_{FUS}}$	280.8
19	$\frac{\partial D_{FUS}}{\partial \alpha_{FUS}}$	-84.23
24	$\frac{\partial L_T}{\partial u_T}$	-.1251
25	$\frac{\partial D_T}{\partial u_T}$	.1931
26	$\frac{\partial L_T}{\partial \alpha_T}$	3430
27	$\frac{\partial D_T}{\partial \alpha_T}$	-7.07
28	$\frac{\partial \alpha_{FUS}}{\partial u} = \frac{\partial \alpha_T}{\partial u}$	.000513
29	$\frac{\partial \alpha_{FUS}}{\partial \alpha} = \frac{\partial \alpha_T}{\partial \alpha}$	.685
32	$\frac{\partial D_F}{\partial u_F}$	3.89

WORKSHEET FOR MUWQ

STORAGE	DERIVATIVE	VALUE
00	D_{TR}	28.3
01	L_F	10060
05	V_0	203
06	1x_F	.5
07	1z_F	-6
08	1x_T	-20
10	$\alpha-\varepsilon$	-.0349
11	$\frac{\partial a_{1F}}{\partial q}$	-.09881
13	$\frac{eb\Omega^2 M_s}{2}$	71821
20	$\frac{\partial M_{HUB_F}}{\partial u_F}$	17.54
21	$\frac{\partial M_{HUB_F}}{\partial \alpha_F}$	17237
22	$\frac{\partial M_{FUS}}{\partial u_{FUS}}$	-36.21
23	$\frac{\partial M_{FUS}}{\partial \alpha_{FUS}}$	48848

STORAGE	DERIVATIVE	VALUE
28	$\frac{\partial \alpha_{FUS}}{\partial u} = \frac{\partial \alpha_T}{\partial u}$	.000513
29	$\frac{\partial \alpha_{FUS}}{\partial \alpha} = \frac{\partial \alpha_T}{\partial \alpha}$	.685
30	$\frac{\partial \alpha_F}{\partial u}$	0
31	$\frac{\partial \alpha_F}{\partial \alpha}$	1
33	$\frac{\partial D_{TR}}{\partial u_{TR}}$	.1545
34	${}^1z_{TR}$	-6
35	x_{u_F}	-3.869
38	x_{w_F}	-18.633
41	z_{u_F}	5.574
43	z_{u_T}	-1.635
44	z_{w_F}	-362.9
46	z_{w_T}	-11.574
50	${}^1x_{TR}$	-30

APPENDIX B
COMPUTER PROGRAMS FOR THE HP-41CV CALCULATOR

These computer programs are written for the HP-41 pocket calculator and are intended to help streamline the process of finding solutions to some of the problems generated by stability and control analysis. Input values should be stored as indicated. When prompted for an input value, the program will store that value in its proper storage register. When the program is executed, the output values will be labeled by alpha characters.

Worksheets are included as an aid to organize the input data for the programs and for the case of the program STAB for recording output data.

A. STAB will calculate many helicopter parameters needed for further calculations. Both main rotor and tail rotor data can be calculated. The equations solved for the output values are listed in the definitions section of Chapter III. Input values and their storage locations are shown in the worksheet for STAB. Space is provided on this worksheet to record the output values. Output values are labeled by alpha characters.

B. XUXW calculates the X-force total stability derivatives. Equations (3-64) through (3-71) are solved with this program. The results are displayed using alphanumerics and are also stored for use in other programs. See the worksheet for XUXW for the input parameters and their proper storage location.

C. ZUZW calculates the total stability derivatives for the Z-direction. Input data is the same as that for XUXW . Output data is listed by alphanumerics and is also stored for future use.

D. MUWQ calculates the total stability derivatives for the pitching moments M_u , M_w , and M_q . A worksheet with input parameters and proper storage locations is also provided for this program. Output data is labeled with alphanumerics.

E. CE finds the coefficients of the characteristic equation generated by the stability determinant, equation (4-9). The program prompts for the input values. It is important to note that the X and Z derivatives must be normalized by aircraft mass and the M derivatives must be normalized by I_y . Also, velocity must be input in units of feet per second. The output data will be the coefficients of the stability quartic, equation (4-9).

01♦LBL "STA	52 XEQ 01
B"	53 RCL 07
02 RCL 04	54 RCL 09
03 RCL 03	55 *
04 *	56 RCL 02
05 STO 14	57 *
06 "OR="	58 RCL 03
07♦LBL 01	59 4
08 ARCL X	60 Y↑X
09 AVIEW	61 *
10 STOP	62 RCL 08
11 RTN	63 /
12 RCL 01	64 "GAMMA="
13 RCL 02	65 XEQ 01
14 *	66 RCL 04
15 RCL 03	67 X↑2
16 /	68 RCL 01
17 PI	69 *
18 /	70 RCL 12
19 STO 00	71 *
20 "SIGMA="	72 RCL 11
21 XEQ 01	73 *
22 RCL 05	74 2
23 RCL 14	75 /
24 /	76 STO 13
25 "MU="	77 "eb="
26 XEQ 01	78 XEQ 01
27 RCL 06	79 RCL 05
28 32.174	80 X↑2
29 /	81 2
30 "MASS="	82 /
31 XEQ 01	83 RCL 07
32 RCL 14	84 *
33 X↑2	85 STO 16
34 RCL 07	86 "Q="
35 *	87 XEQ 01
36 PI	88 RCL 15
37 *	89 RCL 00
38 RCL 03	90 *
39 X↑2	91 STO 17
40 *	92 "TFSIG="
41 STO 15	93 XEQ 01
42 "TF="	94 RCL 17
43 XEQ 01	95 RCL 14
44 RCL 14	96 /
45 RCL 05	97 STO 18
46 +	98 "TFS/OR="
47 "VT="	"
48 XEQ 01	99 XEQ 01
49 RCL 10	100 "END"
50 /	101 "END"
51 "MT="	102 END


```

01♦LBL "XUX
W"
02 RCL 12
03 RCL 09
04 *
05 RCL 32
06 -
07 STO 35
08 RCL 18
09 RCL 10
10 *
11 RCL 19
12 -
13 RCL 28
14 *
15 RCL 17
16 -
17 STO 36
18 RCL 16
19 RCL 10
20 *
21 ST+ 36
22 RCL 26
23 RCL 10
24 *
25 RCL 27
26 -
27 RCL 28
28 *
29 RCL 25
30 -
31 STO 37
32 RCL 24
33 RCL 10
34 *
35 ST+ 37
36 RCL 37
37 RCL 36
38 +
39 RCL 35
40 +
41 "XU="
42♦LBL 01
43 ARCL X
44 RVIEW
45 STOP
46 RTN
47 RCL 14
48 RCL 09
49 *

```

```

50 RCL 15
51 -
52 RCL 01
53 +
54 RCL 05
55 /
56 STO 38
57 RCL 18
58 RCL 03
59 +
60 RCL 10
61 *
62 RCL 19
63 -
64 RCL 02
65 +
66 RCL 29
67 *
68 RCL 05
69 /
70 STO 39
71 RCL 26
72 RCL 10
73 *
74 RCL 27
75 -
76 RCL 04
77 +
78 RCL 29
79 *
80 RCL 05
81 /
82 STO 40
83 RCL 39
84 +
85 RCL 38
86 +
87 "XW="
88 XEQ 01
89 "END"
90 END

```



```

01♦LBL "ZUZ
W"
02 RCL 32
03 RCL 09
04 *
05 RCL 12
06 +
07 CHS
08 STO 41
09 RCL 17
10 RCL 10
11 *
12 RCL 16
13 +
14 STO 42
15 RCL 19
16 RCL 10
17 *
18 RCL 03
19 +
20 RCL 18
21 +
22 RCL 28
23 *
24 ST+ 42
25 RCL 42
26 CHS
27 STO 42
28 RCL 26
29 RCL 28
30 *
31 RCL 24
32 +
33 CHS
34 STO 43
35 RCL 42
36 +

```

```

37 RCL 41
38 +
39 "ZU="
40♦LBL 01
41 ARCL X
42 AVIEW
43 STOP
44 RTN
45 RCL 14
46 RCL 05
47 /
48 CHS
49 STO 44
50 RCL 18
51 RCL 29
52 *
53 RCL 05
54 /
55 CHS
56 STO 45
57 RCL 26
58 RCL 29
59 *
60 RCL 05
61 /
62 CHS
63 STO 46
64 RCL 45
65 +
66 RCL 44
67 +
68 "ZW="
69 XEQ 01
70 "END"
71 END

```


01♦LBL "CE"	
02 "XU=?"	
03♦LBL 01	
04 PROMPT	
05 RTN	
06 STO 01	
07 "XW=?"	
08 XEQ 01	
09 STO 02	
10 "ZU=?"	
11 XEQ 01	
12 STO 03	
13 "ZW=?"	
14 XEQ 01	
15 STO 04	
16 "MU=?"	
17 XEQ 01	
18 STO 05	
19 "MW=?"	
20 XEQ 01	
21 STO 06	
22 "MQ=?"	
23 XEQ 01	
24 STO 07	
25 "VEL=?"	
26 XEQ 01	
27 STO 08	
28 RCL 01	
29 RCL 04	
30 +	
31 RCL 07	
32 +	
33 CHS	
34 "B="	
35♦LBL 02	
36 ARCL X	
37 AVIEW	
38 STOP	
39 RTN	
40 RCL 04	
41 RCL 01	
42 +	
43 RCL 07	
44 *	
45 STO 09	
46 RCL 04	
47 RCL 01	
48 *	
49 ST+ 09	
50 RCL 06	
51 RCL 08	
52 *	
53 ST- 09	
54 RCL 02	
55 RCL 03	
56 *	
57 ST- 09	
58 RCL 09	
59 "C="	
60 XEQ 02	
61 RCL 08	
62 RCL 02	
63 *	
64 CHS	
65 32.2	
66 +	
67 RCL 05	
68 *	
69 STO 10	
70 RCL 08	
71 RCL 06	
72 *	
73 RCL 01	
74 *	
75 ST+ 10	
76 RCL 01	
77 RCL 07	
78 *	
79 RCL 04	
80 *	
81 ST- 10	
82 RCL 02	
83 RCL 03	
84 *	
85 RCL 07	
86 *	
87 ST+ 10	
88 RCL 10	
89 "D="	
90 XEQ 02	
91 RCL 03	
92 RCL 06	
93 *	
94 STO 11	
95 RCL 05	
96 RCL 04	
97 *	
98 ST- 11	
99 RCL 11	
100 32.2	
101 *	
102 "E="	
103 ARCL X	
104 AVIEW	
105 "END"	
106 END	


```

01♦LBL "MUW
Q"
02 RCL 35
03 RCL 07
04 *
05 STO 47
06 RCL 41
07 RCL 06
08 *
09 ST- 47
10 RCL 43
11 RCL 08
12 *
13 ST- 47
14 RCL 33
15 RCL 34
16 *
17 ST- 47
18 RCL 33
19 RCL 10
20 *
21 RCL 50
22 *
23 ST+ 47
24 RCL 23
25 RCL 28
26 *
27 RCL 22
28 +
29 ST+ 47
30 RCL 21
31 RCL 30
32 *
33 RCL 20
34 +
35 ST+ 47
36 RCL 47
37 "MU="
38♦LBL 01
39 ARCL X
40 RVIEW
41 STOP
42 RTN
43 RCL 38
44 RCL 07
45 *
46 STO 48
47 RCL 44
48 RCL 06
49 *
50 ST- 48
51 RCL 46
52 RCL 08
53 *

```

```

54 ST- 48
55 RCL 00
56 RCL 05
57 /
58 RCL 50
59 *
60 ST+ 48
61 RCL 23
62 RCL 29
63 *
64 RCL 21
65 RCL 31
66 *
67 +
68 RCL 05
69 /
70 ST+ 48
71 RCL 48
72 "MW="
73 XEQ 01
74 RCL 08
75 X↑2
76 RCL 46
77 *
78 STO 49
79 RCL 07
80 RCL 11
81 *
82 RCL 01
83 *
84 ST- 49
85 RCL 20
86 RCL 07
87 *
88 ST+ 49
89 RCL 21
90 RCL 31
91 *
92 RCL 06
93 *
94 RCL 05
95 /
96 ST- 49
97 RCL 13
98 RCL 11
99 *
100 ST+ 49
101 RCL 49
102 "MQ="
103 XEQ 01
104 "END"
105 END

```


WORKSHEET FOR STAB

STORAGE	PARAMETER	VALUE
01	b	
02	c	
03	R	
04	Ω	
05	V	
06	W	
07	ρ	
08	I_b	
09	a	
10	V_S	
11	M_S	
12	e	

STORAGE	PARAMETER	VALUE
	ΩR	
	$\sigma = \frac{bc}{\pi R}$	
	$\mu = \frac{v}{\Omega R}$	
	$m = \frac{W}{32.2}$	
	T.F	
	γ	
	$V_T = \Omega R + V$	
	$M_T = \frac{V_T}{V_S}$	
	$\frac{eb\Omega^2 M_S}{2}$	
	$q_0 = \frac{1}{2}\rho V^2$	
	T.F. σ	
	$\frac{T.F.\sigma}{\Omega R}$	

WORKSHEET FOR XUXW

STORAGE	DERIVATIVE	VALUE
01	L_F	
02	L_{FUS}	
03	D_{FUS}	
04	L_T	
05	V_0	
09	α	
10	$\alpha^{-\varepsilon}$	
12	$\frac{\partial L_F}{\partial u_F}$	
14	$\frac{\partial L_F}{\partial \alpha_F}$	
15	$\frac{\partial D_F}{\partial \alpha_F}$	
16	$\frac{\partial L_{FUS}}{\partial u_{FUS}}$	
17	$\frac{\partial D_{FUS}}{\partial u_{FUS}}$	

STORAGE	DERIVATIVE	VALUE
18	$\frac{\partial L_{FUS}}{\partial \alpha_{FUS}}$	
19	$\frac{\partial D_{FUS}}{\partial \alpha_{FUS}}$	
24	$\frac{\partial L_T}{\partial u_T}$	
25	$\frac{\partial D_T}{\partial u_T}$	
26	$\frac{\partial L_T}{\partial \alpha_T}$	
27	$\frac{\partial D_T}{\partial \alpha_T}$	
28	$\frac{\partial \alpha_{FUS}}{\partial u} = \frac{\partial \alpha_T}{\partial u}$	
29	$\frac{\partial \alpha_{FUS}}{\partial \alpha} = \frac{\partial \alpha_T}{\partial \alpha}$	
32	$\frac{\partial D_F}{\partial u_F}$	

WORKSHEET FOR MUWQ

STORAGE	DERIVATIVE	VALUE
00	D_{TR}	
01	L_F	
05	V_0	
06	1X_F	
07	1Z_F	
08	1X_T	
10	$\alpha-\varepsilon$	
11	$\frac{\partial a_{1F}}{\partial q}$	
13	$\frac{eb\Omega^2 M_s}{2}$	
20	$\frac{\partial M_{HUB_F}}{\partial u_F}$	
21	$\frac{\partial M_{HUB_F}}{\partial \alpha_F}$	
22	$\frac{\partial M_{FUS}}{\partial u_{FUS}}$	
23	$\frac{\partial M_{FUS}}{\partial \alpha_{FUS}}$	

STORAGE	DERIVATIVE	VALUE
28	$\frac{\partial \alpha_{FUS}}{\partial u} = \frac{\partial \alpha_T}{\partial u}$	
29	$\frac{\partial \alpha_{FUS}}{\partial \alpha} = \frac{\partial \alpha_T}{\partial \alpha}$	
30	$\frac{\partial \alpha_F}{\partial u}$	
31	$\frac{\partial \alpha_F}{\partial \alpha}$	
33	$\frac{\partial D_{TR}}{\partial u_{TR}}$	
34	${}^1Z_{TR}$	
35	X_{u_F}	
38	X_{w_F}	
41	Z_{u_F}	
43	Z_{u_T}	
44	Z_{w_F}	
46	Z_{w_T}	
50	${}^1X_{TR}$	

LIST OF REFERENCES

1. Hurt, H., Aerodynamics for Naval Aviators, NAVIAR 00-80T-80, Revised January 1965.
2. Meriam, J. L., Dynamics, SI Version, 2nd Ed., John Wiley and Sons, Inc., 1975.
3. Liptrot, R. and Woods, J., Rotorcraft, 1st Ed., Butterworths Scientific Publications, 1955.
4. McRuer, D., Ashkenas, I., and Graham, D., Aircraft Dynamics and Automatic Control, 1st Ed., Princeton University Press, 1973.
5. Kisielowske, E., Perlmutter, A., and Tang, J., Stability and Control Handbook for Helicopters, TR 67-63, USAARM DL, Fort Eustis, VA., August 1967.
6. Tanner, W., Charts for Estimating Rotary Wing Performance in Hover and at High Forward Speeds, NASA CR-114. November 1964.
7. USAF Stability and Control Handbook (DATCOM), Flight Control Division, Air Force Flight Dynamics Laboratory, October 1960.
8. Bramwell, A., Helicopter Dynamics, John Wiley and Sons, Inc., 1st Ed., 1976.
9. Seckel, E., Stability and Control of Helicopters and Airplanes, 1st Ed., Academic Press Inc., 1964.
10. Johnson, W., Helicopter Theory, 1st Ed., Princeton University Press, 1980.
11. Heffley, R., et al, A Compilation and Analysis of Helicopter Handling Qualities Data, NASA CR 3144, August 1979.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3. Department Chairman, Code 67 Department of Aeronautical Engineering Naval Postgraduate School Monterey, California 93940	1
4. Professor Donald M. Layton, Code 67-LN Department of Aeronautical Engineering Naval Postgraduate School Monterey, California 93940	5
5. Professor H. W. Burden, Code 67-ZX Department of Aeronautical Engineering Naval Postgraduate School Monterey, California 93940	1 -
6. Mr. and Mrs. John F. McDonough 80 Rutgers Avenue Bridgeton, New Jersey 08302	2
7. Mr. and Mrs. William J. Pollock 5940 42nd Avenue North St. Petersburg, Florida 33709	2
8. Capt. Robert G. Cramer, Jr., USA C/O Col. R. G. Cramer, USA, Ret. 4903 Meadowbrook N.W. Lawton, Oklahoma 73505	1
9. Maj. John P. Cress, USMC 9 Patrick Street Jacksonville, North Carolina 28540	1
10. Maj. Paul J. Fardink, USA R.D. 2 Fardink Road, Box 636 Ashville, New York 14710	1

11. Lt. Steven R. Laabs, USN 1
SMC Box 1082
Naval Postgraduate School
Monterey, California 93940
12. Lcdr. Thomas R. Nadeau, USN, Code 67 1
Department of Aeronautical Engineering
Naval Postgraduate School
Monterey, California 93940
13. Capt. Joseph H. Schmid, USMC 1
Code 034 ZG
Naval Postgraduate School
Monterey, California 93940

T Thesis

C 05785 O'Neill

C c.1

Calculation of the
longitudinal stability
derivatives and modes
of motion for heli-
copter aircraft.

22 APR 86

5 FEB 87

19 JUL 88

199547

33423

5449
32209

Thesis

05785 O'Neill

c.1

Calculation of the
longitudinal stability
derivatives and modes
of motion for heli-
copter aircraft.

199547

thesO5785

Calculation of the longitudinal stabilit



3 2768 001 97340 7

DUDLEY KNOX LIBRARY